Determination of the Shape (k), Scale (c) Parameters and the Wind Power Density for a Selected Site in Juja

C. Saoke, J. N. Kamau, R. Kinyua

Abstract— Wind is a variable resource in both time and space; it fluctuates so much that it is important to find a model that describes its variation. This is significant in estimating the amount of energy earnings as well as to optimize the design of the wind turbine. This work assesses the wind speed distribution by measuring and analyzing the wind speeds at two heights in Juja. The wind speed data of the site is collected and analyzed on the basis of three months measured hourly time series data. The Weibull distribution model provided a good fit for the recorded wind data. The model is then used to calculate the Weibull distribution parameters for the speed variations as the Weibull scale parameter *(c)* = 5.69 m/s and the Weibull shape parameter *(k)* = 2.06. These parameters are used to estimate the wind power density of the site which was found to be 131.35 W/m^2. Wind rose diagrams depict a large calm condition with a large frequency of wind blowing from the North , north east while frequency are lowest from the south and south east directions.

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Keywords— Power, Scale parameter, Shape parameter, Weibull, Wind speed.

I. Introduction

The continuing rise in population and the shift of nations to industrialization has lead to the increase in energy demand. However factors such as energy sustainability and the gradually emerging consciousness about environmental degradation have provoked priority to the use of renewable alternative sources such as solar and wind energies [1-3]. Wind energy which one of the renewable sources is key to a clean energy future that first used more than 3,500 years ago in boats to transport goods in Egypt and to grind seeds to produce flour. Its advantages such as cleanliness, abundance of resource, low cost, sustainability, safety and effectiveness in job creation has facilitated its fast growth as an energy source during most of the 1990s, expanding at an annual rate from 25% to 35% [3]. According to the wind energy outlook report 2007, wind energy is now one of the most cost effective methods for electricity generation available, the technology is continuously being improved for both cheaper and more productive wind turbines and with this, it can therefore be expected that wind energy will become even more economically competitive in the coming decades [2]. In Kenya, this expansion rate has not yet been realized despite the fact that Kenya is compatible with current wind technology. The main issue is the limited knowledge on the Kenya wind resource. The meteorological station data are quite inadequate since these stations are only 35 spread all over the country [7], [8], [9]. There is significant potential to use wind energy for grid connected wind farms, isolated grids (through wind-solar hybrid systems) and off-grid community electricity and water pumping.

The equatorial areas are assumed to have poor to medium wind resource. This might be expected to be the general pattern for Kenya. However some topography specifics (channeling and hill effects due to the presence of the Rift Valley and various mountain and highland areas) have endowed Kenya with some excellent wind regime areas. The North West of the country (Marsabit and Turkana districts) and the edges of the Rift Valley are the two large windiest areas (average wind speeds above 9m/s at 50 m high). The coast is also a place of interest though the wind resource is expected to be lower about 5-7 m/s at 50 m high [4].

Describing wind speed distribution using the Weibull model

Due to the intermittency of wind resource, It is very important in the wind industry to be able to describe the spatial and temporal variation of wind speeds. This is most useful for turbine designers who need the information to optimize the design of their turbines with a view of minimizing generating costs. On the other hand, turbine investors need the information to estimate their income from electricity generation. A number of studies in the recent years have investigated the fitting of specific distribution to wind speeds for use in practical applications as estimation of wind loads on building and power analysis. The two-parameter Weibull distribution is one of the mathematical models used to study wind data. It is a probability distribution function used to describe wind speed variations and its expressed mathematically by the probability $f(v)$ of observing wind speed v given as in equation 1: [6].

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

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While the corresponding cumulative distribution is

$$F(v) = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{2}$$

Where;

$v \geq 0$ is the wind speed (m/s for this study), $k > 0$ is a shape parameter (dimensionless), $c > 0$ is a scale parameter (m/s)

The variance of a data set from the Weibull distribution is given by equation 3;

$$\delta^2 = \bar{V}\left(\frac{\Gamma\left(1+\frac{2}{k}\right)}{\Gamma^2\left(1+\frac{2}{k}\right)} - 1\right) \tag{3}$$

Further simplification of equation 3 by Justus, (1978) and Lysen, (1983) gives the formulars for k and c as in equation 4 and 5

$$k = \left(\frac{\delta}{\bar{V}}\right)^{-1.086} \tag{4}$$

$$c = \bar{V}\left(0.568 + \frac{0.433}{k}\right)^{-\frac{1}{k}} \tag{5}$$

These parameters are very important in wind energy prediction, k for instance shows how peaked the distribution is, meaning it determines the shape of the Weibull distribution and hence it determines the wind speed variation. Other methods that can be employed to evaluate these parameters include; the maximum likelihood method, having a good fit for the discrete cumulative frequency through the log-log method. Other methods can be obtained from [9].

The power density

Using values of the Weibull scale and shape parameters the wind power density of a given site can be calculated from the wind speed data. The wind power density per unit area analyzed based on a Weibull probability density function is given by:

$$P_w = \frac{1}{2}\rho c^3 \Gamma\left(1+\frac{3}{k}\right) \tag{6}$$

Meaning multiplying the power of each wind speed with the probability of each wind speed obtained from the Weibull curve, then the results gives the distribution of wind energy at different wind speeds (the power density). Out of this power value (equation 6), the maximum extractable power is subject to the Beltz limit given by 59.8% of power density [5], [7], [9].

II. MATERIALS AND METHODS

The objectives of this paper is to use measured data to determine the Weibull shape parameter k and the scale parameter c (m/s) and further use them to estimate the localized wind power density of the site. The wind data studied is collected using two Davis Vantage Pro weather stations located within Jomo Kenyatta University of Agriculture and Technology (JKUAT) in Juja, located 35 kilometers from the Kenyan capital Nairobi. The experimental set is as in figure 1;

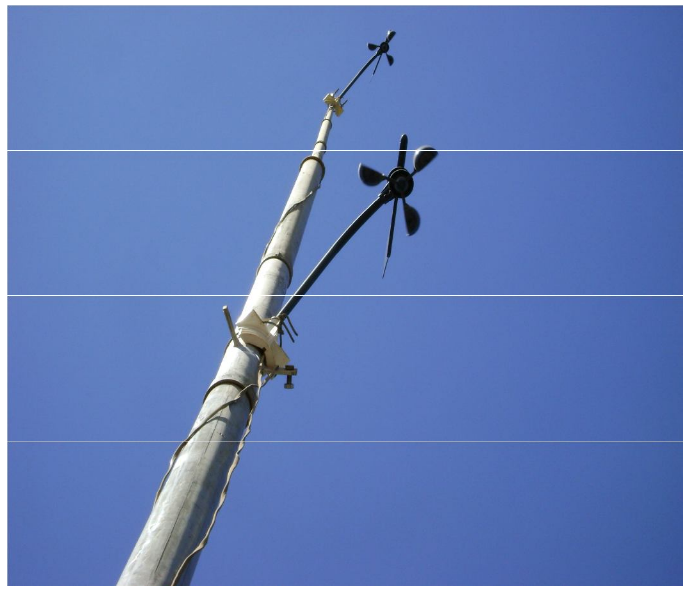

Fig. 1: The set up of equipments.

Wind speeds are measured for a period of three months during the months of September, October and November 2010. This is based on an hourly series daily at heights of 13 m and 20 m. The data is averaged so as to determine the monthly average wind speeds.

The data results are grouped with class width of 1 m/s, the grouped data is then used to statistically model a Weibull distribution which is finally used to estimate the wind power density of Juja.

The wind directions are analysed using the Windographer software to produce the wind rose diagrams as presented in figures 5 and 6.

III. RESULTS

Table 1: The distribution of wind speeds for statistical analysis

Velocity Classes (m/s)	Velocity mid points	Freq.	(fv)	Deviation (D)	fD^2

	(v) m/s	(f)			
0.0-0.9	0.45	91	40.95	-4.59	1917.19
1.0-1.9	1.45	86	124.7	-3.59	1108.37
2.0-2.9	2.45	282	690.9	-2.59	1891.68
3.0-3.9	3.45	325	1121.3	-1.59	821.63
4.0-4.9	4.45	324	1441.8	-0.59	112.78
5.0-5.9	5.45	194	1057.3	0.41	32.61
6.0-6.9	6.45	195	1257.8	1.41	387.67
7.0-7.9	7.45	209	1557.1	2.41	1213.89
8.0-8.9	8.45	152	1284.4	3.41	1767.47
9.0-9.9	9.45	90	850.5	4.41	1750.32
10.0-10.9	10.45	38	397.1	5.41	1112.18
11.0-11.9	11.45	15	171.75	6.41	616.32
12.0-12.9	12.45	5	62.25	7.41	274.54
13.0-13.9	13.45	3	40.35	8.41	212.18
14.0-14.9	14.45	2	28.9	9.41	177.09
15.0-15.9	15.45	1	15.45	10.41	108.36
SUM		**2012**	**10142**		**13504.4**

Table 2: Daily averages of wind speed, air pressure, direction and temperature for September 2010

Date	Height (13 m)		Heigh (20 m)		Temp (^oC)	Direction
Sept 2010	Speed (m/s)	Air pressure (mb)	Speed (m/s)	Air pressure (mb)		
1	3.5	1007.2	3.6	848.6	18	110
2	3.67	1007	3.72	848.1	18	115
3	3.42	1006.9	3.64	847.2	20	105
4	4.2	1007.1	4.26	846	18	100
5	5.14	1007.7	5.4	845	20	130
7	5.08	1007.8	5.48	844.1	21	10
8	3.8	1008.3	4.22	844.6	19	5
9	3.99	1008.6	4.15	844.9	18	360
10	4.74	1008.5	4.84	845.6	18	25
13	5.37	1008.7	5.61	846.3	18	40
14	4.63	1009.1	4.81	847.1	19	40
15	4.14	1009.3	4.91	847.9	20	40
16	3.99	1009.6	4.27	848.3	20	40
17	3.62	1009.8	4.08	848.1	20	40
20	3.85	1010.6	4.23	847.6	20	35
21	3.92	1010.8	4.09	847.1	19	35
22	4.81	1011.1	5.24	846.9	18	15
23	4.25	1011.3	4.38	846.7	19	15
24	4.78	1011.7	5.35	847.2	20	15
25	5.03	1011.6	5.39	847.9	22	30
27	4.89	1011.7	5.38	848.6	20	25
28	6.47	1011.8	6.72	849.2	20	340
29	6.39	1012.1	6.76	849.4	21	310
30	4.28	1012.2	4.75	849.4	20	315
Av.	**4.42**	**1009.54**	**4.72**	**847.04**	**19.4**	
Max.	**6.47**	**1012.2**	**6.76**	**849.4**	**22**	
Min.	**2.61**	**1006.9**	**2.87**	**844.1**	**18**	

Table 3: Daily averages of wind speed, air pressure, direction and temperature for October 2010

Date	Height (13 m)		Height (20 m)		Temp. (^oC)	Direction
October 2010	Speed (m/s)	Air pressure (mb)	Speed (m/s)	Air Pressure (mb)		
1	4.37	1006.3	4.75	848.5	20	120
2	4.42	1006.1	4.86	847.7	20	100
4	4.83	1006.1	5.54	846.7	21	140
5	5.25	1006.1	5.47	845.4	21	45
6	4.8	1006.6	5.14	844.4	20	30
7	4.62	1006.5	5.05	844	19	350
8	4.92	1006.5	5.16	844	21	60
11	5.5	1006.4	5.28	844.2	22	10
12	5.06	1006.9	6.02	844.7	23	5
13	5.48	1007.2	6.44	845.2	22	350
14	6.48	1007.3	6.93	846.2	23	345
15	7.67	1007	7.88	847.1	22	45
18	6.38	1007.3	6.75	847.8	22	30
19	6.18	1007.3	6.49	848.2	22	55
20	6.1	1007.8	6.49	848.2	21	35
21	6.53	1007.6	6.91	847.7	19	60
22	5.85	1008.7	6.12	847.1	18	40
23	6.11	1008.9	6.49	846.8	20	15
25	4.83	1008.8	5.18	846.8	18	320
26	6.1	1008.4	6.49	846.8	19	55
27	6.1	1008.5	6.49	847.2	19	55
28	6.11	1008.3	6.49	847.6	18	85
29	4.58	1008.5	4.82	848.1	19	250
30	5.71	1008.7	5.9	848.3	21	110
Av.	**5.58**	**1007.41**	**5.96**	**846.61**	**20.4**	
Max.	**7.67**	**1008.9**	**7.88**	**848.5**	**23**	
Min.	**4.37**	**1006.1**	**4.75**	**844**	**18**	

Table 4: Daily averages of wind speed, air pressure, direction and temperature for November 2010

Date	**Height (13 m)**		**Height (20 m)**		**Temp (°C)**	**Direction**
Nov. 2010	Speed (m/s)	Air pressure (mb)	Speed (m/s)	Air pressure (mb)		
1	5.34	1007.2	6	848.6	18	115
2	4.09	1007	4.56	848.1	18	105
3	4.3	1006.9	4.82	847.2	20	100
4	3.62	1007.1	4.05	846	18	130
5	4.09	1007.7	4.48	845	20	25
6	3.86	1007.9	4.3	844.4	20	10
7	3.74	1007.8	4.17	844.1	21	5
8	4.56	1008.3	4.95	844.6	19	360
9	4.82	1008.6	5.21	844.9	18	25
10	4.05	1008.5	4.39	845.6	18	40
11	4.48	1008.7	4.79	846.3	18	40
12	4.3	1009.1	4.6	847.1	19	40
13	4.17	1009.3	4.43	847.9	20	40
14	4.26	1009.6	4.59	848.3	20	40
15	4.67	1009.8	5.03	848.1	20	35
16	4.68	1010.6	5.07	847.6	20	35
17	5.15	1010.8	5.24	847.1	19	15
18	4.8	1011.1	4.96	846.9	18	15
19	5.44	1011.3	5.57	846.7	19	15
20	5.9	1011.7	6.05	847.2	20	30
22	4.39	1011.6	5.91	847.9	22	25
23	5.08	1011.7	5.36	848.6	20	340
24	4.46	1011.8	4.86	849.2	20	310
26	4.9	1011.9	5.15	849.1	19	300
27	4.72	1012.1	4.92	849.3	21	315
28	3.99	1012	4	849.3	20	20
29	4.05	1012	4.3	850	19	25
30	4.57	1012.1	4.65	850.1	20	30
Av.	**4.52**	**1009.78**	**4.98**	**847.29**	**19.3**	
Max.	**5.9**	**1012.1**	**6.05**	**850.1**	**22**	
Min.	**3.62**	**1006.9**	**4.05**	**844.1**	**18**	

IV. DISCUSSION

Frequency distribution of wind speeds

The frequency distribution curve of the grouped data is presented in figures 2.

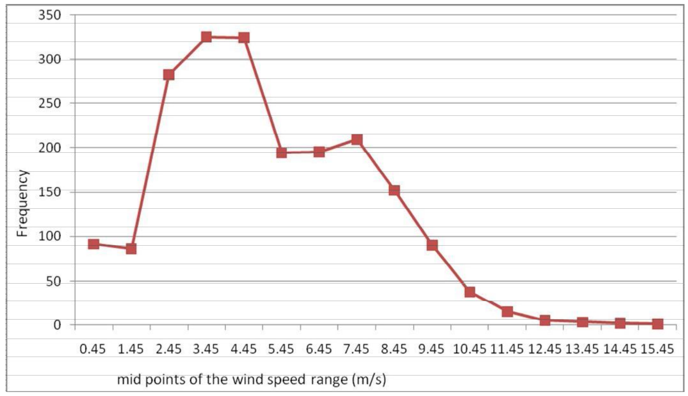

Fig. 2: The frequency distribution of wind speeds at the site

The frequency distribution graph of wind speed is unevenly distributed and shows that the highest number of recorded wind speed values lie within the wind speed range of 3.0-3.9 m/s while the lowest number of recorded wind speed values lie within the range of 15 m/s and above.

The mean wind speed

$$\bar{V} = \frac{\sum f_i v_i}{\sum f_i} = \frac{10142.4}{2012} = 5.04 m/s \qquad (7)$$

The standard deviation is given by:

$$\delta^2 = \frac{\sum f_i D_i^2}{\sum f_i} = \frac{13504.36}{2012} = 6.71$$ where

D is the deviation from the mean wind speed.

Hence the standard deviation is

$$\delta = \sqrt{\delta^2} = \sqrt{6.71} = 2.59 \qquad (8)$$

Because of the intemitent nature of wind speeds to assume and use the mean wind speed only to predict the wind energy potential generally gives very rough estimates. Consequently, for effective energy prediction the wind speed data is fitted in to Weibull distribution model (which is characterised by Weibull scale factor (*c*) and the shape parameter (*k*) for much better predictions.

The Weibull (shape) k*-parameter and the Weibull (scale)* c*-parameter*

The shape parameter was calculated using equation 4, and the value found to be 2.06.The value obtained indicates a peaked distribution i.e, the wind speeds tend to be very close indication less speed variation. This is evident in the Weibull distribution curve in figure 3. The empirical formula in equation 5 was used to calculate the scale parameter which was found to be 5.69 m/s. The calculated Weibull shape and scale parameters gave a Weibull probability curve as shown in figure 3.

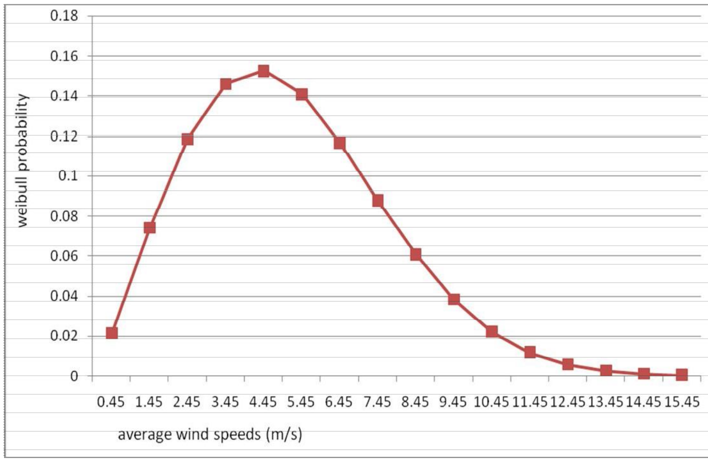

Fig. 3: Weibull probability distribution of the wind speeds

Compared to the frequency distribution graph in figure 2, the Weibull probability curve in figure 3 gives a good fit for this experimental data and therefore provides the probability of obtaining a given wind speed at any given time in the study site.

The area under the Weibull curve in figure 3 equals unity since the probability that the wind will be blowing at some wind speed including zero must be 100 per cent.

As can be seen from the graph, the distribution of wind speeds is skewed to the left showing that there is high probability of wind blowing at a wind speed lower than the median of the horizontal scale. (to the left of the median value) The curve peaks at wind speeds of 3.5 m/s meaning 3.5 m/s are the most probable speeds. Figure 4 gives the cumulative distribution function of the wind speeds with the curve representing the probability that the wind speed will be smaller, greater than or equal to a given wind speed. It is observed that 30% of the wind speeds recorded will be above the wind speed 3 m/s. This result is important especially when analyzing the percentage wind speed distribution for any given site.

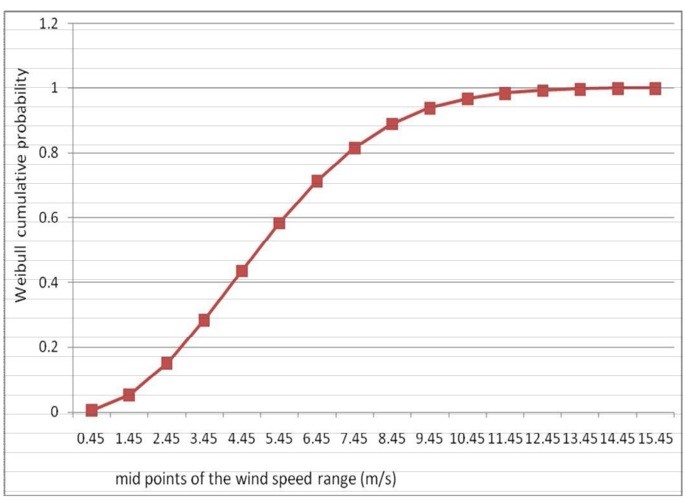

Fig. 4: The Weibull cumulative frequency curve

Estimation of the Mean Wind Power Density

From the formula in equation 6, and using Weibull k=2.06, Scale parameter of 5.69 m/s and air density of 1.1 kg/m^3, the power density was found to be 131.35 W/m^2 . It is observed that the power density is almost twice as much power as the wind would have if it were blowing constantly at the mean wind speed of 5.04 m/s. This is because the power density is obtained from the contribution of every speed with which the wind flows during the fluctuations, while the power available in the wind assumes that the wind constantly blows at a constant speed, this is not practical as sometimes the wind will be extremely high while other times it will be lower. For a height of between 10m to 30m which is the range within which this research was done, the estimated wind power density lies in the wind power density class 2

Considering the effect of the Beltz limit whose value is 0.593, the maximum extractable power approximates 59.3% of the theoretical power density. For this site, the maximum extractable power by a system of unit area operating at its optimum efficiency would be 77.9 W/m^2

Considering the typical wind turbine design for Ngong, the Vestas V52-850 kW wind turbine with the following specifications (www.thewindpower.net);

Table 5: Technical characteristics of Vestas V52 wind turbine

Cut in Wind speed	3 m/s
Cut out wind speed	25 m/s
Rated Power	850 kW
Rotor diameter	25 m
Rotor swept area	2,123.72 m^2
Number of blades	3

If a wind turbine of the above specifications is installed at our site in Juja and operates at optimum efficiency, then from the maximum extractable wind power density for Juja (77.9 W/m^2); the amount of power that can be produced is 165 kW. This value is about a fifth of the production in Ngong station. Juja therefore has a potential for not only wind power utility for local community consumption but also has the capacity to feed considerable amount of power to the national grid if fully harnessed.

Wind Direction Analysis

The Juja wind rose diagram for the heights of 13 m and 20 m are presented in figures 5 and 6 respectively. The horizontal scales represent the frequency of occurrence of wind blowing from a particular direction sector.

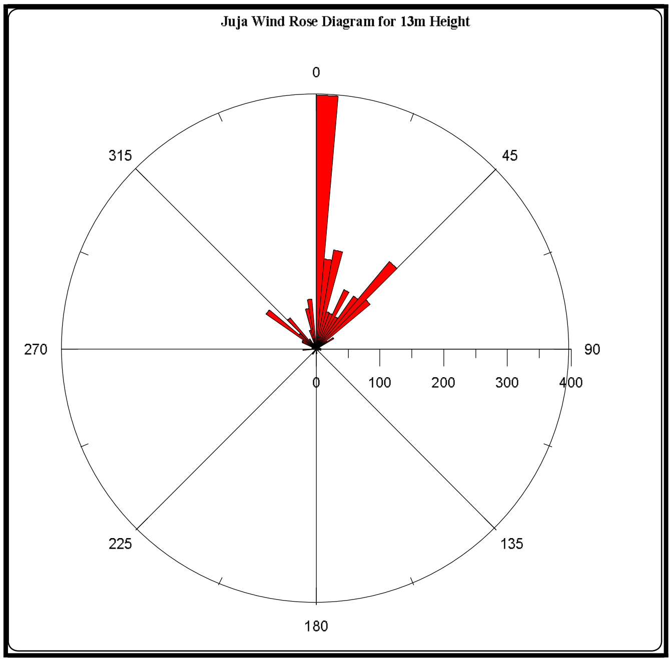

Fig. 5: Wind Rose diagram for the Juja wind speeds at 13 m height

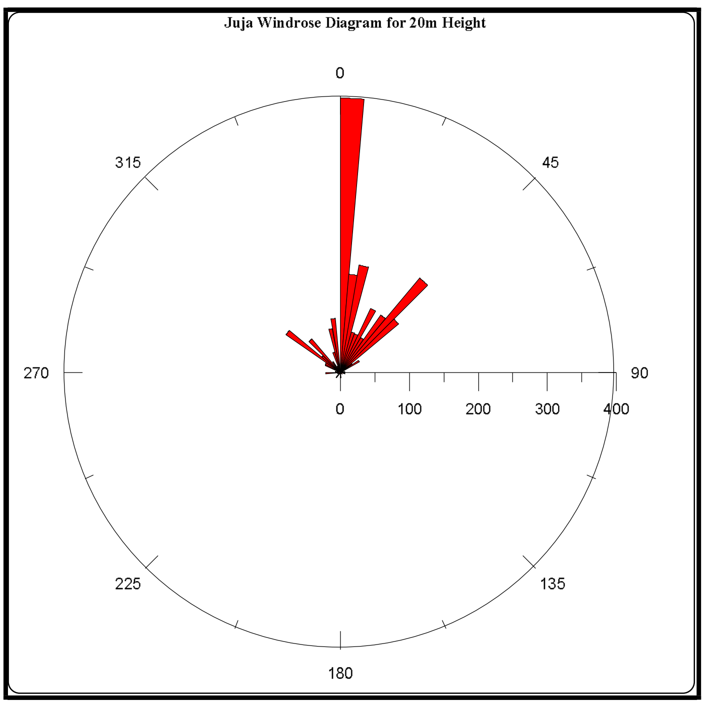

Fig. 6: Wind Rose diagram for the Juja wind speeds at 20 m height

The wind rose diagrams shows the frequency with which wind blows from the different directions; The various arms radiate from the innermost point and the length of each arm represents the total percentage frequency of time (number of occurrence of wind in that particular direction) the wind blows from the direction concerned.

From the diagrams, it is evident that a large frequency of wind at this site blows from the North , north east. On the other hand the frequency of wind is lowest from the south and south east directions; while for the rest of the directions the frequency is generaly low too. There is a large calm condition in the wind system of the station such that the maximum bulge of wind blows from the north to north east diretion It can therefore be said that the station is in the monsoon wind system although there could have also been the influence of the buildings that were closer especially on the southern direction this could have influence the wind direction, moreover, the presence of the dam on the north north east direction could also have influence the regime of wind direction through the breezes.

V. CONCLUSSION

The following conclusions are made from the results of this study:

1. The Weibull shape parameter k for the site was found to be 2.06 while the scale parameter c was found to be 5.69 m/s this depicts that the site has good wind.
2. The most prevalent wind speeds were found to be in the range of 3.0 to 3.9 m/s speeds.
3. From the wind rose analysis, most winds for the site were found to be in the North and North East directions
4. The site is found capable of power generation to the tune of 165 kW if a typical wind turbine like Vestas is to be used.

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