

Wind Converters Theory

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Abstract—From the viewpoint of energy conversion, the most important properties of the wind at a particular location are the velocity of the airstream and the air density. The air density varies with altitude and with atmospheric conditions such as temperature, pressure, and humidity. At sea level and at standard atmospheric temperature and pressure, the value is:

$$\rho = 1.201 \text{ kg/m}^3 \text{ at 1bar or 101.3 kilo Pascal (kPa) pressure and temperature 293K.}$$

In the UK, a useful figure for the atmospheric air density is:

$$\rho = 1.29 \text{ kg/m}^3 \dots \dots \dots (1)$$

In the USA, a commonly quoted figure, for sea level under dry conditions

at a temperature of 0°C (273 K), is:

$$\rho = 1.275 \text{ kg/m}^3 \dots \dots \dots (2)$$

There is a lot of local variation of the values of the air density in different areas of the world. It is found that the temperature, pressure, and density of the air decrease with altitude. For wind-turbine applications, the range of interest is mostly within a couple of hundred feet above ground level. Within this range, it is adequate to use the density values above.

Keywords—Energy Converters, Kinetic energy, Upstream and downstream, Betz Law.

I. INTRODUCTION

Wind energy provided the motive power for sailing ships for thousands of years, until the age of steam. The fortunes of the European colonial powers such as England, France, Germany, Spain, Portugal, Holland and Belgium rested on their mastery of the sea and its navigation. But the intermittent nature and uncertain availability of the wind combined with the relative slowness of wind powered vessels gradually gave way to fossil-fuel powered commercial shipping. Today, most shipping uses oil fuelled diesel engines. However, yachting and small boat sailing remain important recreational sports throughout the world.

The wind has also been used for thousands of years to provide the motive power for machines acting as water pumps or used to mill or grind grain. Such machines came to be known as “windmills”. The operators of windmills in feudal England took the name of their craft and acquired the surname Miller (or Millar).

Wind energy is transmitted by what is essentially a low density fluid (i.e., the wind). The physical dimensions of any

device used to convert its kinetic energy into a usable form are necessarily large in relation to the power produced. Wind availability is not only intermittent but non predictable. The energy source, however, is free, environmentally clean and infinitely renewable. There is no pollution and no direct use of fossil fuels in the energy gathering process.

II. BACKGROUND

In 1891, Professor Poul La Cour in Denmark developed the world first wind-powered electricity generating system to employ aerodynamic design principles. The machines incorporated low-solidity, four-blade rotors 23m in diameter that were designed with primitive aerofoil sections. By about 1920, the use of 25-kW, high-speed wind generators was common in Denmark. As in America, however, cheaper fossil-fuel steam plants put the wind-powered electricity generation industry out of business for many years.

In Germany, Professor Ulrich Hutter developed a series of advanced horizontal-axis designs that utilised aerofoil-type fiberglass and plastic blades with variable pitch, using diameters as large as 35m. The two-blade propeller design sought to reduce bearing and structural failures by the techniques of load shedding.

The first use of a large windmill to generate electricity in the USA was a system built in Cleveland, Ohio in 1888 by Charles F. Brush. Brush’s machine had a multi-bladed rotor, 17 metres in diameter, with a large fan tail. It was the first wind machine to incorporate a step-up gearbox (gear ratio 50:1) and drove a direct-current (DC) generator at 500 rpm. The generator

was rated at 12kW and operated for 20 years.³ By 1920, the two dominant rotor configurations, fan-type and sail-type, had both been found to be inadequate for generating appreciable amounts of electricity. The further development of wind-powered electrical systems in the USA was inspired by the design of aeroplane propellers and aerofoil wing sections. By the mid-1920s, small (1–3 kW) wind-powered DC generators were in widespread use in rural areas, driven by low-maintenance three-blade propeller turbines mounted on tall (e.g., 21m) towers. The predominant companies in the USA were Parris-Dunn and Jacobs Wind-Electric.

These were gradually forced out of business by the customer demand for larger amounts of grid (utility) supplied electricity. The escalating price of oil in the latter years of the twentieth century has caused a comeback and modern successors of the early machines can now be seen all over the country.

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The first successful large wind turbine was the Smith-Putnam machine built at Grandpa's Knob, Vermont, USA in 1941. This privately funded venture proved to be the prototype and inspiration for what has now become a vast industry. As with all horizontal axis propeller machines, the generator and gearbox were mounted on the turbine shaft in housing at the top of the tower. The Smith-Putnam machine was a two-blade propeller downstream system, rated at 1.25MW, with a blade diameter of 53.34m. It operated at a constant rotational speed of 28 rpm. For 35 years, it held the record as the world's largest wind machine system. The electric generator was a synchronous machine that fed electrical power directly into the Central Vermont Public Service Corporation electricity grid. The Smith-Putnam machine suffered two mechanical failures. After a main bearing replacement, there was a spar failure causing one of the propeller blades to fly off. The operating company decided that a repair would be uneconomical and the venture was closed down in 1945.

The largest wind turbine now (2010) operating, very successfully, in the British Isles is situated in Orkney. Located off the north coast of Scotland, the Orkney Islands are in one of the windiest locations in the world, with average wind speeds of the order 17 m/s. The high wind speed permits the 3-MW upstream machine to use the relatively small blade diameter of about 60m at a hub height of about 46m. Electrical power generated by the Orkney machine (Fig. 3.11) is used in the islands and replaces the expensive diesel-electric generation previously used. Wind speeds up to 60 mph can be utilised. The excessively windy location means that the machine is expected to generate more electrical energy (up to 9,000 MWh/year) than any other known existing wind installation.

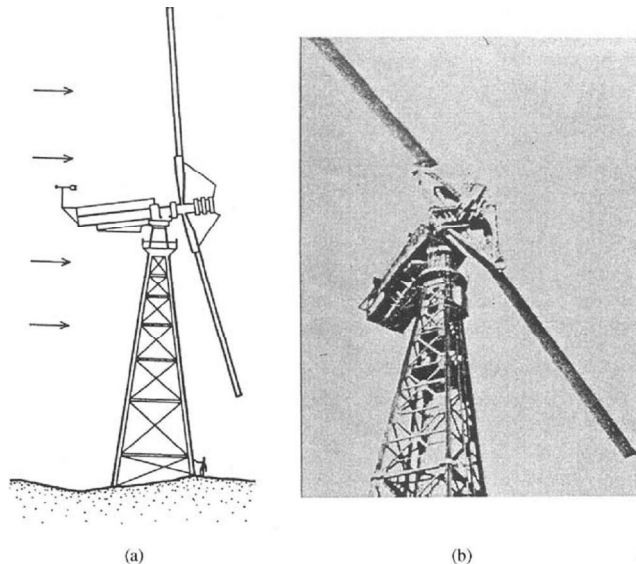


Fig. 1 Smith-Putnam wind machine[4] (a) Diagrammatic, (b) On-site photograph



Fig. 2 3MW wind machine in Orkney, Scotland

III. POWER AND ENERGY

It is important to note that power P and energy W are not the same thing. The energy of a system is its capacity for doing work, irrespective of the time taken to do it. The power of a system is the time rate of doing work or expending energy and therefore has the dimension of energy (or work) divided by time.

$$\text{Power} = \text{Energy (Work)} / \text{Time} \dots \dots \dots (3)$$

For large increments of time t , the average power P is given by:

$$P = \frac{W}{t} \dots \dots \dots (4)$$

For small increments of time dt , the instantaneous value of the power P_{inst} is given by:

$$P_{inst} = \frac{dW}{dt} \dots \dots \dots (5)$$

In most wind-energy calculations, the average power in Eq. (4) is used. In the System International (S.I.), the unit of energy is the joule and the unit of power is the joule per sec (J/s), which is usually called the watt (W). For practical engineering purposes, it is often more convenient to use the kilowatt (kW) or megawatt (MW). A table of conversion factors relating power and energy is given as Table 1. Power in watts is not concerned exclusively with electrical engineering. For example, the rotational mechanical power of engines is often expressed in kilowatts. In terms of the human perception of power, it is sometimes helpful to use the old British power unit of horsepower (HP).

$$1 \text{ Horsepower (HP)} = 746 \text{ W} \dots \dots \dots (6)$$

Energy converters with a rotational mechanical output, such as combustion engines and wind turbines, can be rated either in the mechanical units of horsepower or in the electrical units of kilowatts.

TABLE 1
CONVERSION FACTORS IN POWER AND ENERGY

	Unit	Equivalents
Power	1 watt (W)	1 joule/sec (j/s) = 0.001341 HP
	1 kilowatt (kW)	1000W = 1.34 HP
	1 megawatt (MW)	1000,000W = 1340 HP
	1 horsepower(HP)	745.7W
Power density	1 w/m ²	3.6 Kj/m ² /h
Energy	1 Joule	1watt-second (W/s)
	1 kilowatt hour (kWh)	3.6 × 10 ⁶ Joules
	1 megawatt hour (MWh)	3.6 × 10 ⁹ Joules
Pressure	1 Pascal (Pa)	1 N/m ²
	1 bar	10 ⁵ Pa

A. Theoretical Power Available in the Wind

If the air mass is *m* and it moves smoothly with an average velocity *V*, the motion of the air mass has a kinetic energy (KE).

$$KE = \frac{1}{2}MV^2 \dots\dots\dots (7)$$

Consider a smooth and laminar flow of wind passing perpendicularly (normally) through an element of area *A* of any shape, having thickness *x*, shown in Fig. 2.2. The mass *m* of air contained in an element of volume *Xa* is given, in terms of density *ρ*, by:

$$m = \rho Ax \dots\dots\dots (8)$$

Combining Eqs. (7) and (8) gives, for the KE associated with this mass and volume of air:

$$KE = \frac{1}{2}\rho AxV^2 \dots\dots\dots (9)$$

It is seen that RHS of Eq. (9) represents a force (1/2)ρav² multiplied by a distance *x*. Now, the KE passing through the element per unit time is equal to the power rating:

$$P_{element} = \frac{d(KE)}{dt} = \frac{1}{2}\rho AxV^2 \frac{dx}{dt} \dots\dots\dots (10)$$

But the average time rate of change of the displacement, *dx/dt*, is the average wind velocity *V*:

$$\frac{dx}{dt} = V \dots\dots\dots (11)$$

The average power in the wind *P_w* is obtained by combining Eqs. (10) and (11) to give:

$$P_w = \frac{1}{2}\rho AV^3 \dots\dots\dots (12)$$

Equation (12) is the basis of all wind power and energy calculations. The most obvious feature is that the wind power is proportional to the cube of the average wind speed. It is

clear that the average wind speed is, by far, the dominant consideration in wind turbine location.

TABLE 2
POWER AVAILABLE IN THE WIND³ (AT STANDARD TEMPERATURE AND PRESSURE)

Power	Swept area, A	Wind velocity, V	Power equation, P _w
W	M ²	m/s	0.64AV ³

B. Theoretical Maximum Power Extractable from the Wind

Only a fraction of the total theoretical power available in the wind, represented by Eq. (12), is extractable. It is an intrinsic property of all physical systems that when energy is converted from one form to another, this conversion is accompanied by various energy losses. The result is that conversion is always subject to significant intrinsic limitations of efficiency.

Let a flow of smooth and steady air with an upstream average velocity *V₁* impinge upon the rotor of a wind machine. Some of the energy from the wind is transferred to the wind machine rotor so that the smooth and steady air far downstream flows at a smaller average velocity *V₂*. The KE reduction of the airflow, of mass *m*, per unit time is:

$$\begin{aligned} \text{Kinetic Energy}(KE) &= \frac{1}{2}mV_1^2 - \frac{1}{2}mV_2^2 \\ &= m\frac{1}{2}(V_1^2 - V_2^2) \dots\dots\dots (13) \end{aligned}$$

In the process of extracting energy from the wind, the wind velocity *V_r* that actuates the rotor is less than the upstream “free wind” velocity *V₁*. With an ideal and lossless system, all of the energy reduction in the airstream is transferred to the rotor of the wind machine. The downstream average velocity *V₂* is then smaller than the actuating velocity *V_r* at the rotor. Combining Eq. (8) with Eq. (11) for the airstream at the rotor blades gives an expression for the time rate of air mass transferred.

$$\frac{dm}{dt} = \rho A \frac{dx}{dt} = \rho AV_r \dots\dots\dots (14)$$

The power at the rotor is, from Eq. (5), the time rate of KE transferred.

$$P_r = \frac{d(KE)}{dt} \dots\dots\dots (15)$$

Substituting Eqs. (13) and (14) separately into Eq. (15) gives:

$$P_r = \frac{1}{2} \frac{dm}{dt} (V_1^2 - V_2^2) = \frac{1}{2} \rho AV_r (V_1^2 - V_2^2) \dots\dots\dots (16)$$

Now the air mass passing through the rotor undergoes not only an energy reduction but a reduction of linear momentum:

$$\text{Reduction of Linear Momentum} = m(V_1 - V_2) \dots \dots \dots (17)$$

The time rate of the change of momentum reduction is a force, of value:

$$\frac{d}{dt}(m(V_1 - V_2)) = \frac{dm}{dt}(V_1 - V_2) = \rho AV_r(V_1 - V_2). \quad (18)$$

By equating the rate of the change of KE transfer (16) with the power associated with the rate of the change of momentum, from Eq. (18), it is found that the wind velocity V_r at the rotor may be expressed as:

$$V_r = \frac{V_1 + V_2}{2} \dots \dots \dots (19)$$

In this maximize model of airflow, the wind velocity at the rotor is therefore the average of the upstream and downstream steady wind velocities. Substituting for V_r from Eq. (19) into Eq. (16) gives an expression for the power extractable from the wind by the rotor. Using the symbol P_{ex} to denote extracted power,

$$P_{ex} = \frac{1}{4} \rho A (V_1 + V_2) (V_1^2 - V_2^2) = \frac{1}{4} \rho A V_1^3 \left\{ 1 + \frac{V_2}{V_1} \left\{ \frac{V_2}{V_1} \right\}^2 - \left\{ \frac{V_2}{V_1} \right\} \right\}$$

$$= \frac{1}{4} \rho A V_1^3 \left\{ 1 + \frac{V_2}{V_1} \right\} \left\{ 1 - \left\{ \frac{V_2}{V_1} \right\}^2 \right\} \dots \dots \dots (20)$$

The value of wind velocity ratio V_2/V_1 that results in maximum power transfer is calculated by differentiating Eq. (20) with respect to (V_2/V_1) and equating to zero. This results in a quadratic equation showing that to maximize P_{ex} the ratio V_2/V_1 must have the values either $V_2/V_1 = 1/3$ or $V_2/V_1 = -1$.

The negative option is meaningless so that the correct solution is:

$$\frac{V_2}{V_1} = \frac{1}{3} \dots \dots \dots (21)$$

Substituting Eq. (21) into Eq. (20) gives an expression for the maximum possible power extraction, under ideal conditions.

$$P_{ex} = (\max) \frac{8}{27} \rho A V_1^3 = \left\{ \frac{16}{27} \right\} \frac{1}{2} \rho A V_1^3$$

$$= \{0.593\} \frac{1}{2} \rho A V_1^3 \dots \dots \dots (22)$$

The very important result (22) is sometimes referred to as Betz Law, being named after the German astroscientist Albert Betz of Göttingen. This states that “even with ideal wind

energy conversion the maximum power transferable is only 0.593 or 16/27 of the total power in the wind”. In reality, only a fraction of this theoretical maximum power is realised, even by the best-designed turbines.

Table 3 shows the values of the maximum theoretical power obtainable from a range of wind-turbine sizes at typical wind speeds.3 It is notable how large a circular area must be used to generate any useful amount of power. For example, in a 16-km/h wind, which is a light breeze, a swept area of 8.0m diameter would realise only a maximum theoretical value of 1.5Kw

(and a practical value of roughly one-half of that). This immediately points to the difficulty of using wind energy for domestic use in urban areas—the swept area required is too large to be practicable.

TABLE 3
MAXIMUM THEORETICAL POWER EXTRACTABLE BY
IDEAL WIND MACHINE³ $\left[\frac{8}{27} \rho A V^3 \right]$

Wind speed (km/h)	12.5	25.0	50.0	100	200
16.1	0.11	0.45	1.829	7.32	29.27
32.2	0.93	3.75	15.0	59.76	239.02
48.3	3.17	12.68	50.73	203.05	812.19
64.4	7.50	30.0	120.0	479.88	1919.51
80.5	14.6	58.7	235.1	940.55	3762.19
96.6	25.3	101.4	405.8	1623.7	6493.9

CONCLUSIONS

One of the great advantages from the use of wind energy to provide electricity is the consequent reduction of the use of fossil fuel. In addition to the saving of the fuel, the elimination of fossil fuel combustion represents a corresponding reduction in the emission of pollutants. The most polluting of the fossil fuels is coal.

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