Application of Copula Theory in Modelling
Risks by Incorporating Dependence Structure

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A thesis submitted in fulfilment for the Degree of Doctor
of Philosophy in Applied Statistics in the Jomo Kenyatta
University of Agriculture and Technology

2011

ABSTRACT

Insurance companies maintain different lines of business as a mode of diversification which in itself aids in reducing risks of encountering ruin. The dependence structure of these lines of business cannot be ignored especially in rate making as it reduces diversification benefits. However, in practice you would find that risks are more heterogeneous than homogeneous a problem that can be solved by breaking down the risks into a number of homogeneous categories. The lines of business are considered, here, to contain sub-classes which are homogeneous. The lines will depict a hierarchical structure from the sub-classes to the main lines of business up to the portfolio level and their dependence structure is studied here by the hierarchical copulas. In risk classification, similar risks should be assigned to the same class with respect to each variable. The dependencies are examined by fitting copulas, estimating the dependence parameters and lastly using distance matrices to cluster the risks together. The distance to use in the classification is determined by the problem at hand. The empirical study derives its data from the general insurance business in Kenya where the risks are classified by the Copula based approach. This work proposed the use of the upper tail dependence, measured by the tail index, derived from the dependence parameter in determining the retention limits for a re-insurance arrangement. Though the dependence is not the only factor to consider for such reinsurance treaties the forwarding proportions should be somewhere proportional to $1/(1 - \text{Tail index})$. This will ensure that the highly dependent risks in the upper tail will forward higher proportion to the re-insurer and vice versa.