ON THE SPECTRA OF N ¨O RLUND Q AND ALMOST N¨O RLUND Q OPERATORS

JOTHAM RAYMOND AKANGA

A thesis submitted in fulfilment for the Degree of Doctor of Philosophy in Pure Mathematics in the Jomo Kenyatta University of Agriculture and Technology

2007

ABSTRACT

The main objectives in this study are to investigate and determine the spectra of N¨orlund means acting as operators on Banach sequence spaces. We also aim to determine the spectra of almost triangular matrices. Specifically we determine the spectrum of the N¨orlund Q operator on the Banach spaces c0, c, bv0, bv. We also determine the spectrum of an almost N¨orlund Q matrix operator on c0 and c.
In all the cases mentioned we show that the spectrum comprises of the disc centered at the point (1
2 ,0) of radius 1
2 . We also construct the fine spectrum of the Q
operator on c.
Apart from the more obvious benefit i.e., the solution of systems of linear equation of which the spectrum of operators is all about; there are other more subtle, but equally important applications of the research. A central problem in the whole of mathematics and even science and engineering; is the determination of the convergence or non- convergence of sequences and series. Mathematics, especially Mathematical analysis, develops and is maintained via the concept of convergence of sequences and series. Even in applied science and Engineering, one is interested in the convergence of a sequence or a series of results generated during experimentation. Established theorems such as the ratio theorems and integral theorem, are not applicable in a variety of sequences and series. Even where they apply, they just determine convergence and not the limit or sum of the convergent sequence or series. Tauberian theorems in Summability Theory handle this problem well.The convergence and even limit of a convergent sequence or series is determined from the convergence of some transform of it together with a side condition, (Maddox, 1970); (Boos, 2000) pp. 167 - 204; (Hardy, 1948) pp. 148 - 177; (Powell and Shah, 1972) pp. 75 - 92; or (Maddox, 1980) pp. 65 - 80, e.t.c. The spectrum of an operator plays a crucial role in the development of a Tauberian theory for the operator, (Dunford and Schwartz, 1957) pp. 593 - 597. It is evident from the mentioned books that a Tauberian theory for N¨orlund operators is almost non-existent. Therefore we are confident the results
x
developed in the thesis will open a floodgate for such theorems for N¨orlund means. In turn this will find application in diverse fields such as, integral transforms and Fourier analysis; and in probability and statistics through such areas involving central limit theorem, almost sure convergence, summation of random series, Markov chains e.t.c; (Boos, 2000) pp. 256 - 257.
Chapter 1 deals with literature review, a summary of Functional Analysis material; as well as classical summability methods; especially those that are pertinent to our study.
Chapter II deals with the spectrum of the Q matrix on c0 and c. In chapter III we investigate the spectrum of the N¨orlund Q operator on the spaces bv0 and bv. Chapter IV is concerned with the fine spectrum of the Q matrix operator on c. In Chapter V we investigate the spectrum of an almost N¨orlund Q matrix operator on c0 and c. Chapter VI gives an overview of the results obtained and points the way

forward for future research interests.
In achieving the results, we used a combination of classical and modern functional analytic methods as well as Summability methods. Functional analytic methods usually appeal to the powerful Banach space theorems, such as Hahn - Banach; Banach-Steinhaus; exetra. Classical Summability methods employ sequence space mapping theorems such as Silverman - Toeplitz; Kojima - Shur; exetra