Synchronization in a network of Oscillators with Delayed Coupling

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ABSTRACT

The study of coupled oscillators with time lag can get its applications in; Neurobiology, Laser arrays, Microwave devices, Communications satellites and electronic circuits, just to mention but few. That is why we studied a population of *n* oscillators each with an asymptotically stable limit cycle coupled all-to-all by a linear diffusive like path with a time lag, t . The system of equations was inbuilt with symmetries which we exploited to get an analytical understanding of the dynamics of the system. The symmetries then helped us get two *n*-dimensional invariant manifolds: the diagonal manifold and the other orthogonal manifold. We exploited the symmetries in the coupling terms to establish the range of time delay t for stability of synchronized state.

We did a rigorous study of the condition of stability and persistence of the synchronized manifold of diffusively coupled oscillators of linear and planar simple Bravais Lattices by considering *n* (*n* ³ 2) , *d*-dimensional oscillators each with an asymptotically stable limit cycle coupled all-to-all by a nearest neighbor linear diffusive like path. We used the invariant Manifold Theory and Lyapunov exponents to establish the range of coupling strength for stability and robustness of the synchronized manifold. The 4th and 5th order Runge-Kutta method, together with *ode*-45 and *dde*-23 Mat lab solvers were the numerical methods we used to get the numerical solution of our problem. We established the estimate for bound of t for which the synchronized manifold remains stable when the oscillators are coupled in an all-to-all configuration. The synchronized state is seen to be stable when t < 9. Even for significant time delays, a stable synchronized state exists at a very low coupling strength.

From the study we realized that if synchronization exists for a certain coupling

configuration, then there exist a $k0 > 0$ such that for all $k0 > k$, synchronization manifold is stable and persist under perturbation.