CUTTING FORCE CONTROL FOR BALL END MILLING OF SCULPTURED SURFACES USING FUZZY LOGIC CONTROLLER

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Abstract

Productivity and precision in machining is normally limited by the forces emanating from the cutting process. Due to the inherent varying nature of the work piece in terms of geometry and material composition, the peak cutting forces vary from point to point during machining process. In order to increase productivity without compromising on machining accuracy, it is important to control these cutting forces. In this paper a fuzzy logic control algorithm is developed that can be used to limit maximum peak forces in milling of spherical surfaces using ball end mills. The controller can adaptively maximise the feedrate subject to allowable cutting force on the tool, which is very beneficial for a time consuming complex shape machining. This control algorithm is implemented in computer numerical control (CNC) machine. To demonstrate constant peak force control, the radial depth of cut was abruptly changed from 1.0 to 1.5 mm. As a result, the peak cutting forces increased. In order to counteract this increase and maintain a constant peak force, the pre-programmed table feed was decreased by assigning the appropriate override. For the above change in depth of cut, the override was found to be about 78%, which is a drop of table feed from 30 mm/min to 23.4 mm/min. It has also been demonstrated that the controller developed can provide stable machining and improve the performance of the CNC milling process by varying feedrate in real-time.

Key words: Ball end milling, sculptured surfaces, productivity, fuzzy logic controller, federate

1.0 Introduction

Ball mills are widely used in the machining of free-form surfaces such as those encountered in dies/moulds and aircraft structural components. Machining of complex surfaces in these sectors has identified difficulties such as the geometrical complexity of the parts requiring close tolerances and high surface integrity.

Cutting forces are the main factor governing machining accuracy, surface quality, machine tool vibration, power requirements and tool life and hence the ability to predict them is useful for the design of machine tool structure and cutting tools as well as for the control and optimization of machining processes (Ikua *et al.,* 2001). In fact, it may give information about cutter deflections which lead to dimensional errors, machine tool chatter and tool breakage. Thus tool life and surface integrity can be optimized in selecting appropriate cutting conditions.

When a CNC machine programmer is writing part program, machining parameters such as feedrate, spindle speed and cutting depth are programmed off-line depending on his/her experience or stored machine database (Cus *et al*., 2006). Even with machining data in database, variations in material structure such as different hardness make it necessary to set these parameters extremely conservatively to avoid tool breakage or excessive tool wear and as a result, the machine is usually operated far below the optimum operating conditions.

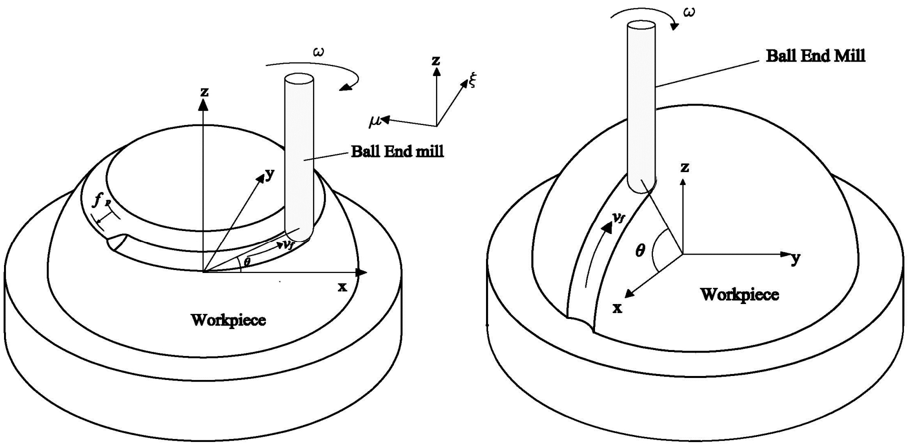
To ensure quality surface integrity of products, reduce the machining cost and increase the machining efficiency, it is necessary to adjust the machining parameters during machining process in real-time in order to justify the optimal machining criteria (Cus *et al*., 2006). Classical and modern control theories have successfully been used in areas where the systems are well defined either deterministically or stochastically, but they cannot cope with the needs of manufacturing industries because of the complexity and vagueness of practical processes (Kim *et al*., 2000). Fuzzy control and fuzzy systems have the benefit of replicating all desired features of human input, while maintaining all the advantages of classical and modern control theories. Control rules are presented in a form of IF-THEN Statements (Ming *et al*., 2002).

In this paper, an adaptive fuzzy logic controller for regulation of feedrate was developed based on the emanating cutting forces in machining of spherical surfaces.

1.1 Modelling of Cutting Forces

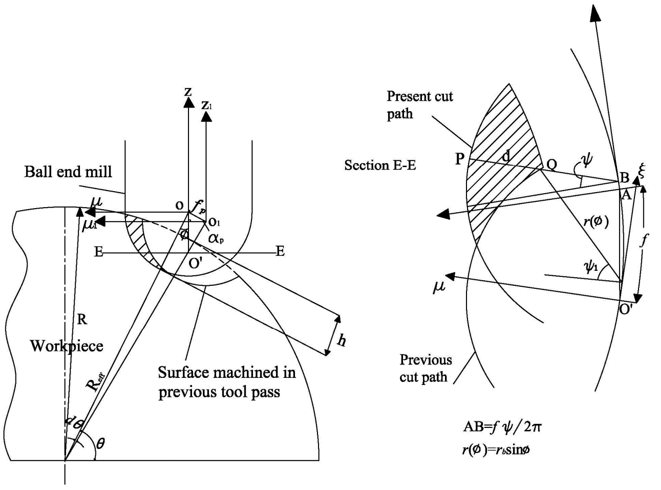
1.1.1 Model for Ball End Milling

The cutting styles for ball end milling of spherical surfaces are shown in Fig. 1. In this paper two cutting styles are considered i.e., contouring and ramping. In the contouring, the tool cuts along the latitude of the sphere while in the ramping; the cutting is along the longitude. As shown in Fig. 2, f_p is the cross-feed and θ is the milling position angle. Different cutting modes, which are named with reference to the direction of cross-feed and tool-feed, are considered.

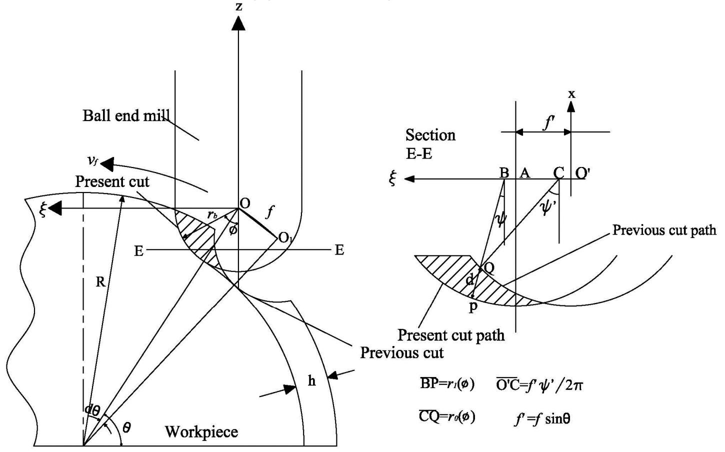


(a) Contouring (b) Ramping

Figure 1: Various cutting styles and cutting modes



(a) Contouring



(b) Ramping

Figure 2: Cutting models for spherical surfaces

In contouring cutting style, the cross-feed is referred to as “Up cross-feed” if the z-component of the tool-feed is in the positive z-direction and “Down cross-feed” if it is in the negative z-direction. The feed is referred to as “Up-cut” if the direction of motion of cutting edge due to rotation is the same as that of the tool-feed, and “Down cut” if the direction of motion of the cutting edge due to rotation is opposite that of tool-feed. In ramping, the feed is referred to as “Upward cut” when the z-component of the tool feed is in positive z-direction and “Downward cut” when it is in negative z-direction.

In this paper four cutting modes were considered that is. Up cross-feed, Up-cut (U-U), Up cross-feed, Down cut (U-D), Down cross-feed, Up-cut (D-U) and Down cross-feed, Down cut (D-D). In contouring, a point P on the cutting edge follows a epitrochoidal path, which is defined by the following general parametric equations,

$$\xi_p = r(\phi)\sin\left(\psi - \frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right) + R_{ef}\sin\left(\frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right)$$

$$\mu_p = r(\phi)\cos\left(\psi - \frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right) + R_{ef} - R_{ef}\cos\left(\frac{2\pi n n_t f + f n_t \psi}{2\pi R_{ef}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$n = 1,2,3\ldots$$

f is the feed rate, n_t is the number of cutting edges, n is the number of revolutions, ψ and ϕ are the rotational and locational angles of the point, respectively, and $r(\phi)$ is the radius of the arc traced by the cutting point, $R_{ef} = R_{eff}\cos\theta$, where R_{eff} is the effective radius given by, $R_{eff} = R + r_b - h$, R is the radius of the workpiece, r_b is radius of the ball end mill and *h* is nominal depth of cut.

In “Down cut”, *f* is taken as negative. By considering equations of loci traced in two consecutive cuts for a tool with one cutting edge, obtain the equation

$$r(\phi)\sin\left\{\frac{2\pi R_{ef}-f}{2\pi R_{ef}}\cos^{-1}[G]\right\}+R_{ef}\sin\left\{\frac{f}{2\pi R_{ef}}cos^{-1}[G]\right\}-r(\phi)\sin\left(\psi-\frac{2\pi f+f\psi}{2\pi R_{ef}}\right)$$
$$-R_{ef}\sin\left(\frac{2\pi f+f\psi}{2\pi R_{ef}}\right)-d\sin\left(\frac{2\pi f+f\psi}{2\pi R_{ef}-\psi}\right)=0 \quad \text{....................(2)}$$

where,

$$G=cos\psi-\frac{d^2}{2r(\phi)R_{ef}}+\frac{d}{R_{ef}}-\frac{dcos\psi}{r(\phi)}$$

from which the chip thickness *d* in the horizontal plane can be determined.

1.1.2 Instantaneous Cutting Forces

In this paper, the cutting process is considered as an aggregation of small oblique cuttings along the helical cutting edge of the ball end mill tool. Therefore, oblique cutting theory is used to calculate the cutting forces. The radial and tangential components of differential cutting forces acting on an infinitesimal element of the cutting edge shown in Fig 3, are given by

$$dF_r(\phi,\psi)=(k_r+k_{rc}s(\phi,\psi))r_b d\phi$$
$$dF_r(\phi,\psi)=(k_r+k_{rc}s(\phi,\psi))r_b d\phi \quad \text{....................(3)}$$

where k_r and k_t are the radial and tangential edge force coefficients, which account for ploughing and rubbing, k_{rc} and k_{tc} are radial and tangential cutting coefficients given by (Ming et al., 2002). $s(\phi,\psi)$ is the instantaneous depth of cut.

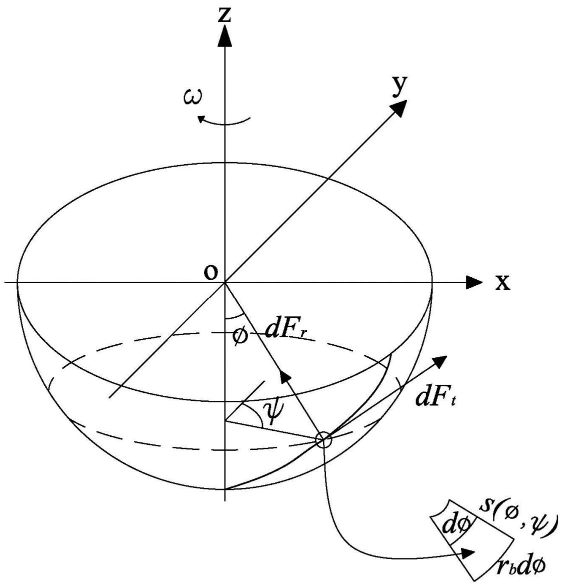


Figure 3: Elemental cutting forces

$$k_{rc}=\frac{\tau_s\sin(\beta-\alpha_e)}{\sin\phi_s\cos(\phi_s+\beta-\alpha_e)}$$
$$k_{rc}=\frac{\tau_s\sin(\beta-\alpha_e)}{\sin\phi_s\cos(\phi_s+\beta-\alpha_e)} \quad \text{....................(4)}$$

In this equation, *τs* is the shearing strength of the workpiece material, *β* is the friction angle, ϕ_s is the shear angle and α_e is the effective rake angle. Due to variation of the helix angle along the cutting edge of the tool, the effective rake angle is calculated for each element of the cutting edge. According to the Stabler’s rule of chip flow (Boothroyd, 1975), the chip flow direction angle is approximately equal to the helix angle. Then the

effective local rake angle can be determined in terms of local helix angle γ and normal rake angle α_n as
$\sin\alpha_e = \sin^2\gamma + cos^2\gamma \sin\alpha_n$ .... (5)

The elemental cutting forces in Eq. (3) are projected to the Cartesian coordinates systems as follows

$$\begin{Bmatrix} dF_x \\ dF_y \\ dF_z \end{Bmatrix} = T \begin{Bmatrix} dF_t \\ dF_r \end{Bmatrix} \qquad (6)$$

where the matrix *T* is given by

$$T = \begin{bmatrix} -\cos\psi & -\sin\phi\sin\psi \\ \sin\psi & -\sin\phi\cos\psi \\ 0 & \cos\phi \end{bmatrix} \qquad (7)$$

The cutting force components are obtained by performing numerical integration of the elemental forces in Eq. (6), for the engaged part of the cutting edge.

1.2 Design of Cutting Force Fuzzy Control System

1.2.1 FLC structure

The proposed fuzzy control system is shown in Fig. 4. In this paper, cutting force control is investigated in contour machining of spherical surfaces and since milling is on the X-Y plane, the cutting force component in the Z-direction is small and stable which in this case can be neglected for constant cutting force control. The cutting forces that were fed to the input of the FLC constituted F_X and F_Y components measured from dynamometer. The proposed controller has four inputs, i.e., force in X-axis (ΔX), change in force error in X-axis $(\Delta^2 X)$, force error in Y-axis (ΔY) and change in force error in Y-axis$(\Delta^2 Y)$. (ΔX) and (ΔY) are computed from the difference of crisp values of the reference and measured cutting forces. That is $\Delta X = F_{XRef} - F_X$ and $\Delta Y = F_{YRef} - F_Y$, while $(\Delta^2 X)$ and $(\Delta^2 Y)$ are computed from the difference in errors for two consecutive sampling intervals.
i.e., $\Delta^2 X = \Delta X(k \mid 1) \quad \Delta X(k)$ and $\Delta^2 Y = \Delta Y(k \mid 1) \quad \Delta Y(k)$. Where *k* is the sampling interval.

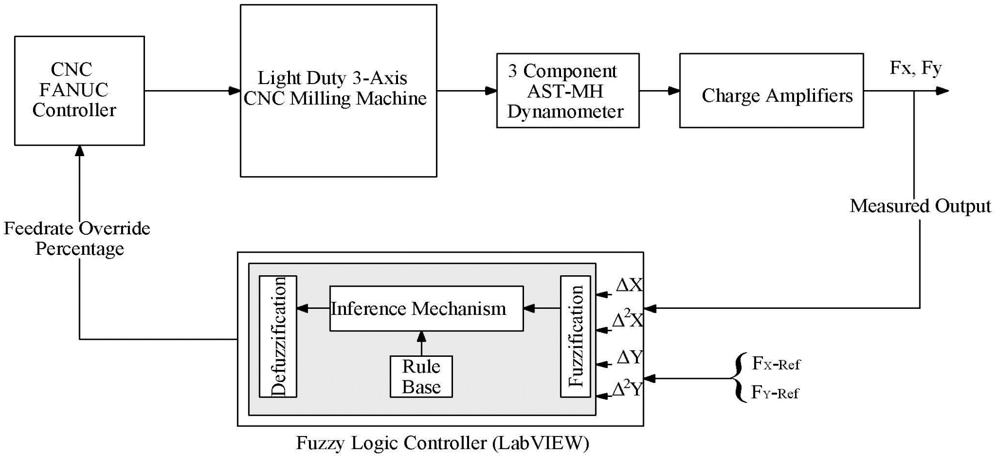


Figure 4: Schematic diagram of fuzzy logic controller for milling process

The design of the fuzzy logic controller (FLC) involves; identification of the inputs and outputs including their ranges, design of the fuzzy membership function for each input and output, construction of the knowledge base that contains the fuzzy rules used to operate the system, fuzzy decision making or inference mechanism that performs fuzzy reasoning and defuzzification to determine the crisp control output. The output of FLC is feed-rate-percentage-override (FRPO). In this paper FRPO varies from 0-150% depending on the error and change in error between the reference and measured instantaneous cutting force.

1.2.3 Membership Functions

The FLC for cutting force control is a typical multiple-input-single-output (MISO) control system with four inputs and one output. The input linguistic variables to the FLC are, force error in X-Axis (ΔX), change in force error in X-axis ($\Delta^2 X$), force error in Y-axis (ΔY) and change in force error in Y-axis ($\Delta^2 Y$). In the design of FLC, the machine operator's intuition and experience were used to define the fuzzy variables. For each input linguistic variable, there are three linguistic terms. These include Negative (Ne), Zero (Ze), and Positive (Po). In defining the fuzzy linguistic terms for the input fuzzy variables, triangular membership functions were considered. Since the aim of controller is to minimize the error between the set and measured force, the range for all input fuzzy variables was set at (-1.5~1.5N). The output of FLC is comprised of FRPO which has five fuzzy linguistic terms, i.e, Negative small (NS), Negative medium (NM), Zero (Ze), Positive medium (PM), Positive large (PL). In defining the linguistic terms for output linguistic variable, trapezoidal membership functions were considered. Figure 5 and 6 shows the membership functions for input and output linguistic variables respectively.

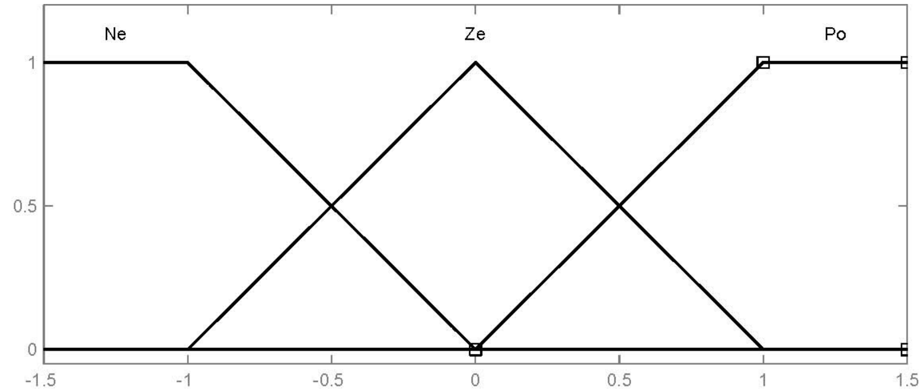


Fig. 5: Membership function for force error in X-Axis

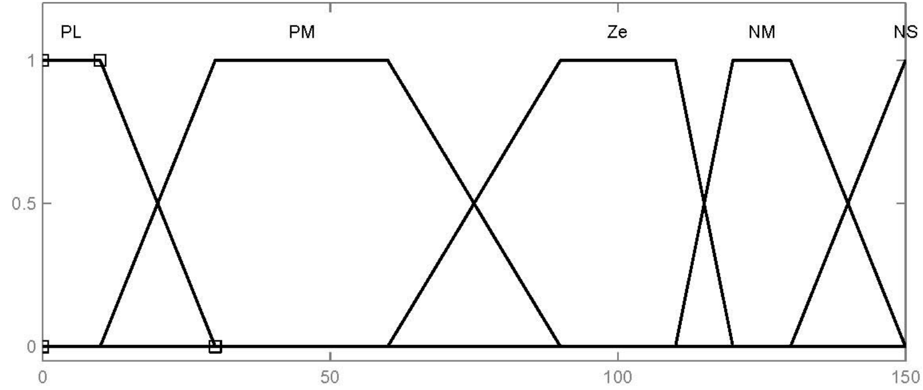


Figure 6: Membership function for Feed rate percentage override

1.2.2 Rule Base

One of the most crucial components of the fuzzy controller structure is the control rule module. The set of control rules defines the system behaviour and replaces the mathematical modelling of the system. The fuzzy rules, which use fuzzy inputs to determine system actions, are obtained from the skilled operators, experiments, and prior knowledge of the end milling processes. The expert knowledge is usually in the form of "IF-THEN" rules, which are easily implemented by fuzzy conditional statements in fuzzy logic. The collection of fuzzy control rules that are expressed as fuzzy conditional statements forms the rule base or the rule set of FLC. In the design of FLC rule base, multiple "IF-THEN" statement are joined by connective AND. In order to maintain cutting force to the optimum value, the error between reference and measured force is minimized as much as possible by varying table feed. The actual table feed is computed by multiplying the pre-programmed table feed by the FRPO. The rules were derived based on force prediction model and are summarized in Table 1.

Table 1: Fuzzy rules for cutting force control

Rules	ΔX	$\Delta^2 X$	ΔY	$\Delta^2 Y$	Feedrate percentage override
1.	Ne	Ne	Ne	Ne	Ns
2.	Ne	Ne	Ne	Ze	Ns
3.	Ne	Ne	Ne	Po	Nm
4.	Ne	Ne	Ze	Ne	Ns
5.	Ne	Ne	Ze	Ze	Nm
6.	Ne	Ne	Ze	Po	Ze
7.	Ne	Ne	Po	Ne	Nm
8.	Ne	Ne	Po	Ze	Ze
9.	Ne	Ne	Po	Po	Ze
⋮	⋮	⋮	⋮	⋮	⋮
81.	Po	Po	Po	Po	PL

1.2.3 Fuzzy Inference Mechanism

Inference mechanism is the procedure for determining the influence produced by the antecedent part of the fuzzy rule on the consequent part of the rule. In this paper the Mamdani inference system was used because of its widespread acceptance. Since fuzzy rules used had multiple antecedents, the fuzzy operator (AND) was used to obtain the firing levels of the rules. The results of the antecedent evaluation were applied to the membership function of the consequent to obtain the overall control action. In order to evaluate the conjunction of the fuzzy rule antecedents, the AND fuzzy operation was used as follows.

$$\mu_{\triangle X} \cap \mu_{\triangle^2 X} \cap \ldots = \min\left(\mu_{\triangle X}, \mu_{\triangle^2 X}, \ldots\right) \qquad \ldots\ldots(8)$$

The centre of gravity (COG) defuzzification method was used to come up with the overall control action which varies the machine table feed depending on the forces generated during the machining process as shown in Figure 7.

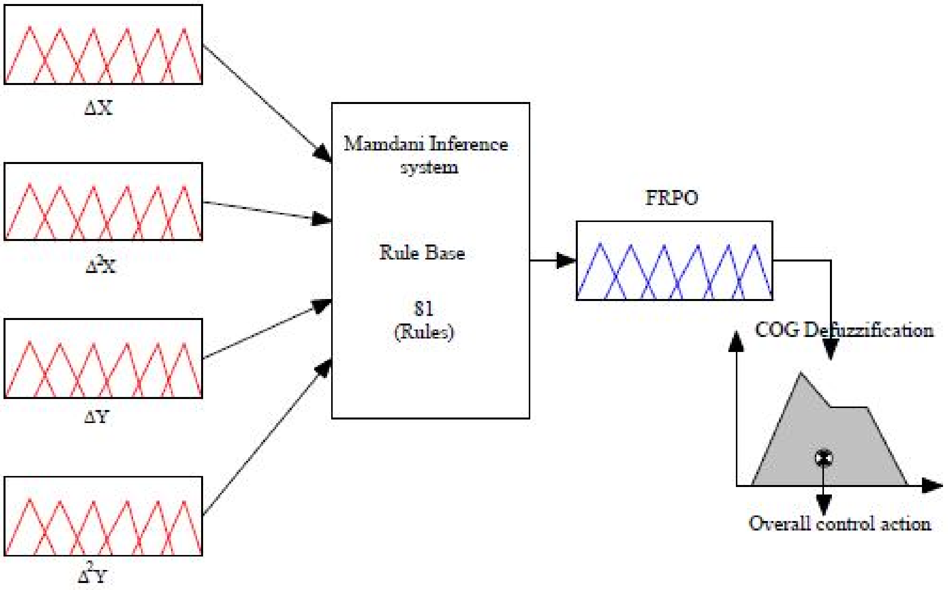
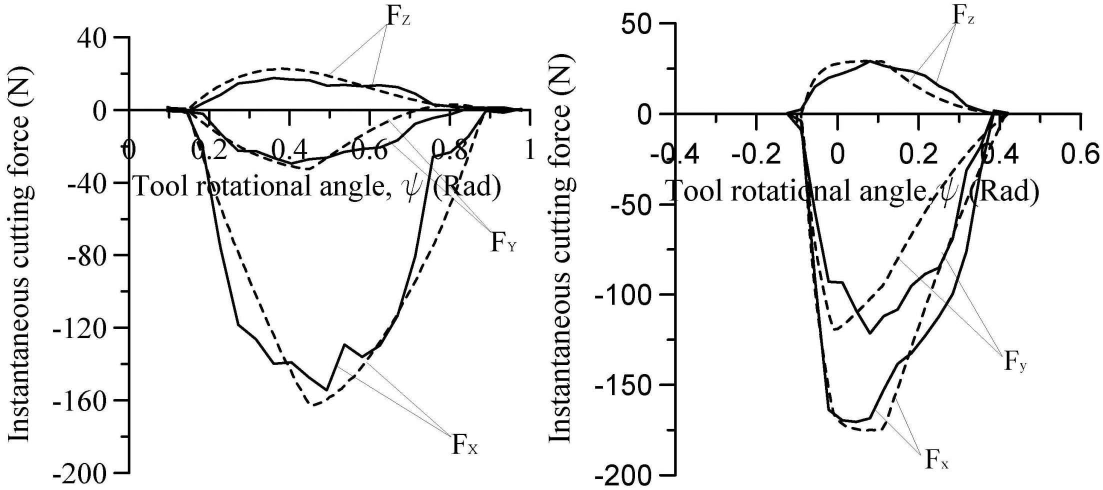


Figure 7: Mamdani Inference system

3.0 Results and Discussion

3.1 Cutting Forces

In designing a fuzzy controller for cutting force control, optimal cutting force from the force prediction model was used as a reference to the FLC. The fuzzy logic controller works in a closed loop to control cutting force by minimizing the error between the measured and reference force. The proposed design of the fuzzy logic controller was implemented and tested using a personal computer in LabVIEW environment. The controller was tested on a precision, light duty 3-axis CNC milling machine (DENFORD TRIAC PC) with FANUC Numerical controller. The cutting force signal in X- and Y-axis components were captured in a computer via a PCI data acquisition card (NI PCI-6259) which was connected to the strain amplifiers via a 68-pin shielded connector block (NI SCB-68). The workpiece material was aluminium Al 6063 T4. Typical waveforms of the predicted and measured instantaneous cutting forces of spherical surfaces for U-U and D-U cutting models at milling position angle $\theta = 45^\circ$ are shown in Fig. 8 (a) and (b). The dotted lines in these figures represent the predicted forces, and continuous lines represent the measured one. It can be shown that in all cutting modes there is a close correspondence between the predicted and measured forces. The captured forces were compared with reference optimum force from the force prediction model to get force error in both *X*- and *Y*-axis components. From the computed force errors, change in force error was calculated using consecutive sampling time. The control algorithm was designed such that the machining process occurred at constant force. This approach is desirable because it increases the machining efficiency by considering the compromise between tool life and material removal rate. The peak cutting forces *Fx* and *Fy* were specified as 120 N and 40 N, respectively. Figure 9 show a photo of experimental set up, Figure 9.



(a) U-U

(b) D-U

Figure 8: Typical waveforms of cutting forces

Figure 9: Experimental set up

3.2 Feedrate Response

Figure 11 shows feedrate response on change in radial depth of cut. To demonstrate constant peak force control, the radial depth of cut was abruptly changed from 1.0 to 1.5 mm. As a result, the peak cutting forces increased and the FRPO from FLC's output was recorded. In order to counteract this increase and maintain a constant peak force, the pre-programmed table feed was decreased by assigning the appropriate override. For the above change in depth of cut, the FRPO was found to be about 78%, which is a drop of table feed from 30 mm/min to 23.4 mm/min as shown in Figure 11.b It can be seen that the peak cutting force was maintained at a constant value of about 120 N despite the change in radial depth of cut.

Figure 10 show the cutting force response when there is no controller applied. When the radial depth of cut is step changed from 1.0 mm to 1.5 mm, the peak cutting force increased. This can be attributed to the fact that machining operation was carried at a constant table feed.

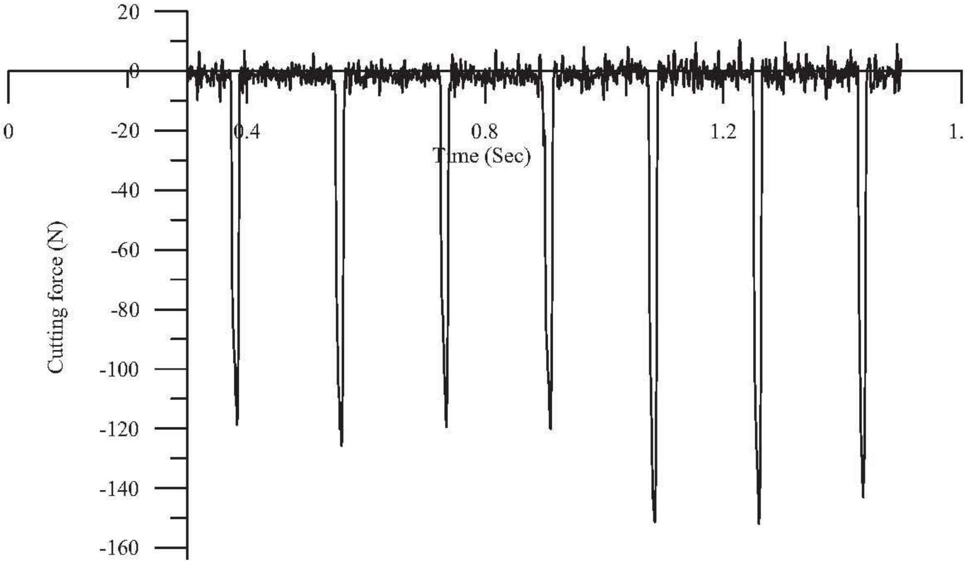
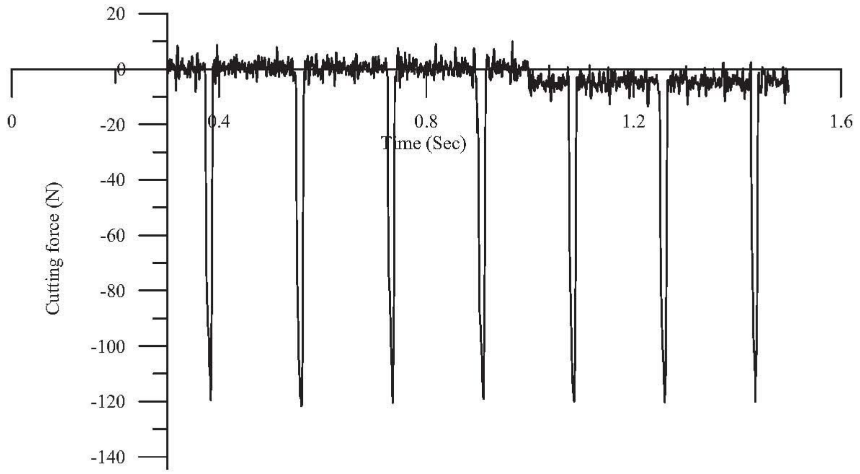
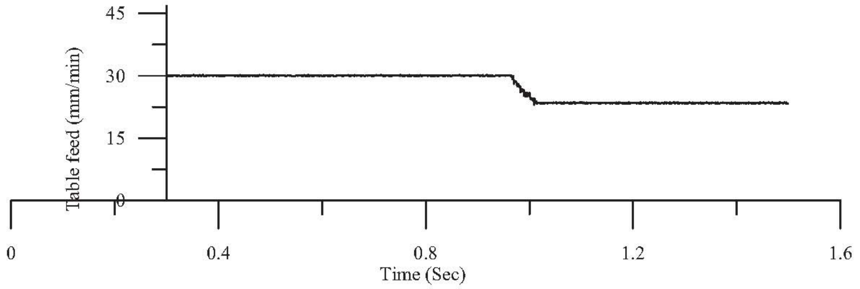


Figure 10: Cutting at a constant feed (30 mm/min)



(a) Cutting force



(b) Feed rate response

Figure 11: Feedrate response on change in radial depth of cut

4.0 Conclusion

In this paper a fuzzy logic control algorithm for controlling the peak cutting forces was developed and implemented in LabVIEW environment. The algorithm was such that the machining occurred at a constant peak force. This had a compromise between the tool life and MRR in that when cutting force exceeded the reference peak force, the controller reduced the table feed thus preventing tool damage. When the force was below the reference force, the controller increased the feedrate hence increasing Material Removal Rate (MMR). A step change in depth of cut from 1.0 mm to 1.5 mm, resulted in a change in table feed from pre-programmed 30 mm/min to about 23.4 mm/min. Conventionally when the NC program is being designed, it is usually based on the maximum depth to select a conservative feedrate for the whole machining process. With the introduction of constant cutting force control loop, feedrate is adjusted in real time based on the feedback to increase the machining efficiency. The proposed controller has an advantage over the conventional controller such as PID in that one does not have to model the plant especially when it come to milling of sculptured surfaces since in such cases most of the parameters do not relate linearly and sometimes are ill defined.

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