CORRELATION OF WIND PATTERNS USING WEIBULL AND RAYLEIGH MODELS FOR ST.XAVIER SECONDARY SCHOOL, NAIVASHA AND JKUAT SITES

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Abstract

The rapidly increasing population and industrialization has led to high energy demand. Juja, having a rapidly growing population and a major University requires an alternative energy source due to frequent power failure. This paper analyzed the wind speed characteristics for purposes of determining wind energy potential in Juja and Naivasha using the heights of 10 m and 30 m and for a period of three months. The wind data was recorded in 10 minutes intervals. Mean diurnal and monthly variations were calculated. The wind speed averages for 10 m and 30 m were 2.54 m/s and 3.04 m/s, respectively. The average wind speeds at the two heights were used to calculate wind shear exponent and roughness parameter for Juja site which were 0.1652 and 0.0374 respectively. Weibull scale and shape parameters were obtained using Weibull-fit, Regression and Maximum LikeliHood. The wind speed distribution was modeled using the Weibull and Rayleigh probability distribution functions. Power densities for different methods were calculated. Results obtained from Juja site were compared with results acquired from Naivasha. The mean power densities for Juja and Naivasha by Weibull model were 12.96 W/m² and 36.52 W/m² respectively. Results by Rayleigh model were 2.82 W/m² and 3.31 W/m². The Weibull-fit and Maximum Likelihood fitted best Juja site and Naivasha respectively. Windographer and Microsoft Excel were used to determine and generate the functions while WindRose diagrams were used to analyze wind direction. The results will guide the designers on selection of appropriate type of wind turbines for optimum power production.

Key words: Power density, wind distribution models, wind direction, wind speed

1.0 Introduction

Growing global population along with fast depleting reserves of fossil fuels is influencing researchers to search for clean and pollution free sources of energy which are sustainable and cost-effective. These types of energy include; solar, wind, waves, tidal and bio-energies. Wind energy is a never ending natural resource which has shown its great potential in combating climatic change while ensuring clean and efficient energy, yet it is the most under exploited energy. Wind turbine technology has led to significant growth of wind power generation across the world. However, wind energy is more sensitive to variations with topography and wind patterns compared to solar energy. It can be harvested economically if the turbines are installed in a windy area and suitable turbine is properly selected. Wind speed forecasting is a critical factor in assessing wind energy potential and performance of wind energy conversion systems. Wind fluctuation demands a necessary model describing its variation thereby estimating the amount of energy as well as to optimize the design of the wind turbine (Kamau *et al*., 2009). The most suitable wind turbine model which needs to be installed in a wind farm is selected by careful wind energy resource evaluation. It is therefore important to choose an accurate distribution model which closely mimics the wind speed distribution at a particular site (Kollu *et al*., 2012). The development and utilization of wind energy in developing countries, particularly Africa, has been hindered by absence of adequate measurements and assessment studies to ascertain its potential viability for power generation (Ajayi *et al*., 2011)

Juja which is at an altitude of 1416 m above sea level; 1^0 10' S, 37^0 7' E about 35 km from Nairobi, having a rapidly growing population and a major University, Jomo Kenyatta University of Agriculture and Technology (JKUAT) requires an alternative energy source due to frequent power failure. Use of power generators during power outage leads to air pollution and hence the need for clean, reliable, sustainable and cost effective sources. Wind energy is among the least exploited with about 25 MW from the two Ngong wind farm phases.

Thorough wind speed analysis is therefore critical. A research based on different methods brings out a better picture of the state of wind in a site. Several methods and Probability Density Functions (PDFs) have been used in literature to describe wind speed characteristics. The methods include Weibull-fit, Regression, Standard deviation, Maximum likelihood, Chi-square among others while PDFs include Weibull, Rayleigh, bimodal Weibull, lognormal, gamma among others (Piefracesco, 2011; Kollu *et al.,* 2012). This has not been done in Juja and Naivasha.

2.0 Materials and Methods

A research was carried out in Juja at 10 m and 30 m heights in order to measure wind speeds and directions at the two heights above the ground. The set-up of apparatus used is indicated in figure 1.

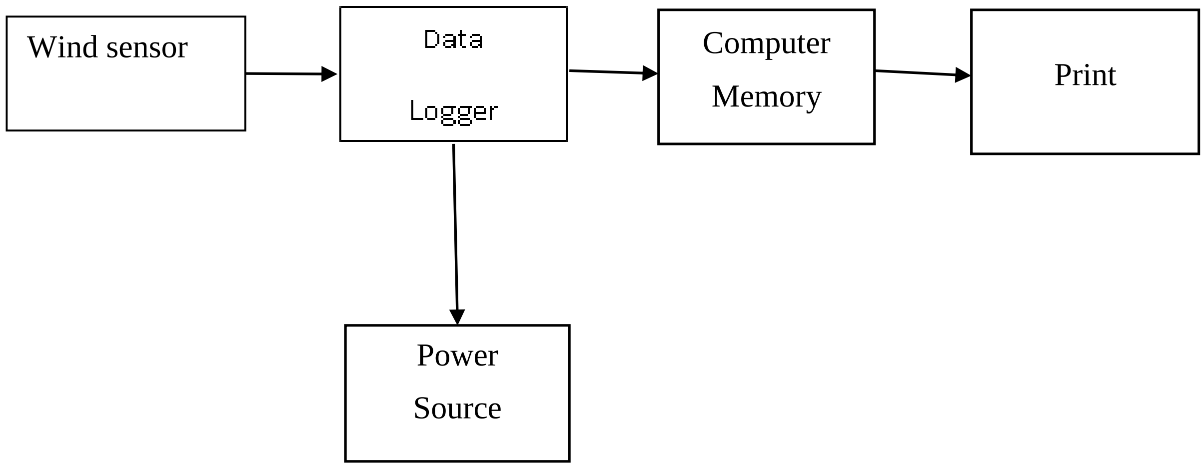

Figure 1: Block diagram of experimental set-up

The wind shear exponent and surface roughness were determined. Weibull scale parameter c and shape parameter k were obtained for Juja and St.Xavier secondary school (BRIGHT PROJECT), Naivasha in order to correlate data from the two sites. Wind energy potential was modeled using Weibull and Rayleigh distributions functions.

The analysis of various parameters is indicated in the following sub-topics;

2.1 Power Available in the Wind

Wind power is extracted from the wind using wind turbines. The power, P, is given by

$$P = \tfrac{1}{2}\rho A v^3 \qquad \text{.......} 1$$

Where;

$\rho = \frac{p}{RT}$; P is air pressure, R gas constant and, T temperature in degrees Kelvin. The wind velocity at the rotor plane is the average of upstream and downstream wind speeds.

Maximum useful power is given by Betz's constant ≈ 0.59.

2.2 Power Law Formula

Wind speed near the ground changes with height; this involves an equation that forecasts wind speed at different height by using the available wind speed data. Kantar and Usta, (2008) stated that the most commonly used equation for the variation of wind speed with height is the power law;

$$v_2 = v_1 \left(\frac{h_2}{h_1}\right)^{\alpha} \qquad \text{.......} 2$$

Where v_1 (m/s) is the actual wind speed recorded at height h_1 (m), and v_2 (m/s) is the wind speed at h_2 (m). The exponent (α) depends on the surface roughness and atmospheric stability.

Roughness length which is used to characterize shear and the height above the ground is not constant (Manwell *et al*., 2009) and thus equation 2 can be modified to yield equation 3.

$$v_2 = v_1 \frac{\ln\left(h_2 / z_0\right)}{\ln\left(h_1 / z_0\right)} \quad \ldots\ldots 3$$

Wind shear exponent, a difference in wind speed and direction vertically has been determined for various types of terrains (Linacre and Geerarts, 1999).

2.3 Wind probability distributions

Two of the commonly used functions for fitting a field data probability distribution in a given location over a certain period of time are the Weibull and Rayleigh distribution models (Kantar and Senoglu, 2008). The Weibull probability density function , $f_R(v)$ is given as;

$$f_R(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} e^{-\left(\frac{v}{c}\right)^k} \quad \ldots\ldots 4$$

The Rayleigh $f_R(v)$ distribution is a special case of the Weibull distribution in which the shape parameter k=2. The probability density functions of the Rayleigh distribution is therefore given by;

$$f_R(v) = \frac{2v}{c^3} e^{-\left(\frac{v}{c}\right)^2} \quad \ldots\ldots 5$$

2.4 Methods of Obtaining Weibull Parameters

2.4.1 Maximum Likelihood (MLH)

The parameters k and c (m/s) can be estimated by using the Maximum Likelihood Method, Zhang *et al*. (2006) and Justus et al. (1978) as;

$$k = \left(\frac{\sum_{i=1}^{n} v_i^k \ln(v_i)}{\sum_{i=1}^{n} v_i^k} - \frac{\sum_{i=1}^{n} \ln(v_i)}{n} \right)^{-1}$$

……………………………………………………………………………..6

$$c = \left(\frac{1}{2} \sum_{i=1}^{n} v_i^k \right)^{\frac{1}{2}} \quad \ldots\ldots 7$$

2.4.2 Weibull-fit

The shape and scale parameters as per Weibull-fit are as follows;

$$c = \frac{v_m}{\Gamma\left(1 + \frac{1}{k}\right)} \quad \ldots\ldots 8$$

$$k = \left(\frac{\sigma_v}{v_m} \right)^{-1.090} \qquad \text{.......9}$$

Where σ_y is standard deviation.

$$v_m = \left(\frac{\sum_{i=1}^{n} f_i v_i}{\sum_{i=1}^{n} f_i} \right) \qquad \text{.......10}$$

$$\sigma_y = \left[\frac{\sum_{i=1}^{n} f_i \left(v_i - v_m \right)^2}{\sum_{i=1}^{n} f_i} \right]^{\frac{1}{2}} \qquad \text{.......11}$$

2.4.3 Regression

The cumulative probability function of the Weibull distribution (Akpinar and Akpinar, 2004), is given by;

$$F(v) = 1 - \exp[-(\frac{v}{c})^k] \qquad \text{.......12}$$

To determine k and c requires a good fit of the equation above to the recorded discrete cumulative frequency function. By taking the natural logarithm of both sides of equation 12 twice gives;

$$\ln\langle -\ln[1 - F(v)] \rangle = k \ln(v) - k \ln c \qquad \text{.......13}$$

Plotting $\ln\langle -\ln[1 - F(v)] \rangle$ against $\ln(v)$ presents a straight line whose gradient is *k* and the y-intercept is $-k \ln c$ (from which c can be calculated).

2.5 Wind Power Density Function

The evaluation of wind power per unit -area is of fundamental importance in assessing wind power projects (Zhou *et al*., 2009). The formula for P_v is given by;

$$P_v = \frac{1}{2n} \sum_{i=1}^{n} \rho \left(v_i^3 \right) \qquad \text{.......14}$$

Where the wind speed at stage i, is v_i , n, the number of non-zero wind data points and ρ, is approximated to $1.225\, kg/m^3$ and depends on altitude, air pressure and temperature. The expected monthly or annual wind power density per unit area of a site based on Weibull probability density function P_W (Akpinar and Akpinar, 2004) can be expressed as follows;

$$P_W = \frac{1}{2}\rho c^3 \Gamma\left(1+\frac{3}{k}\right)$$ ………………………………………………………………………………………………………..15

Where c is the Weibull scale parameter (m/s) given by equation 2.9

2.6 Statistical Analysis of Wind Power Density Based on the Weibull and Rayleigh Models

The two significant parameters k and c are closely related to the mean value of the wind speed v_m . By extracting c from equation 11 and setting k = 2, the power density for the Rayleigh model $\left(P_R\right)$ is found to be;

$$P_R = \frac{3}{\pi}\rho v_m$$ …….16

Where;

$$V_m = c\Gamma\left(\frac{3}{2}\right) = 0.88623c$$ ………………………………………………………………………………………………..17

3.0 Results Analysis and Discussion

Average diurnal wind speeds, wind directions, and temperatures were obtained for three months. The average wind speeds were 2.54 m/s and 3.04m/s and 3.54 m/s for Juja, 10 m and 30 m heights and 10 m at St. Xavier, Naivasha respectively. Wind shear exponent and roughness parameters for Juja were 0.1652 and 0.0374 respectively**.** These parameters are in line with Linacre E. and Geerts B. (1999).The wind shear profile was obtained by use of Windographer (figure 2). Diurnal variation for the whole period of three months is shown by figure 3 while those for comparison between Juja and Naivasha, 10 m, for different months are shown in the appendices.

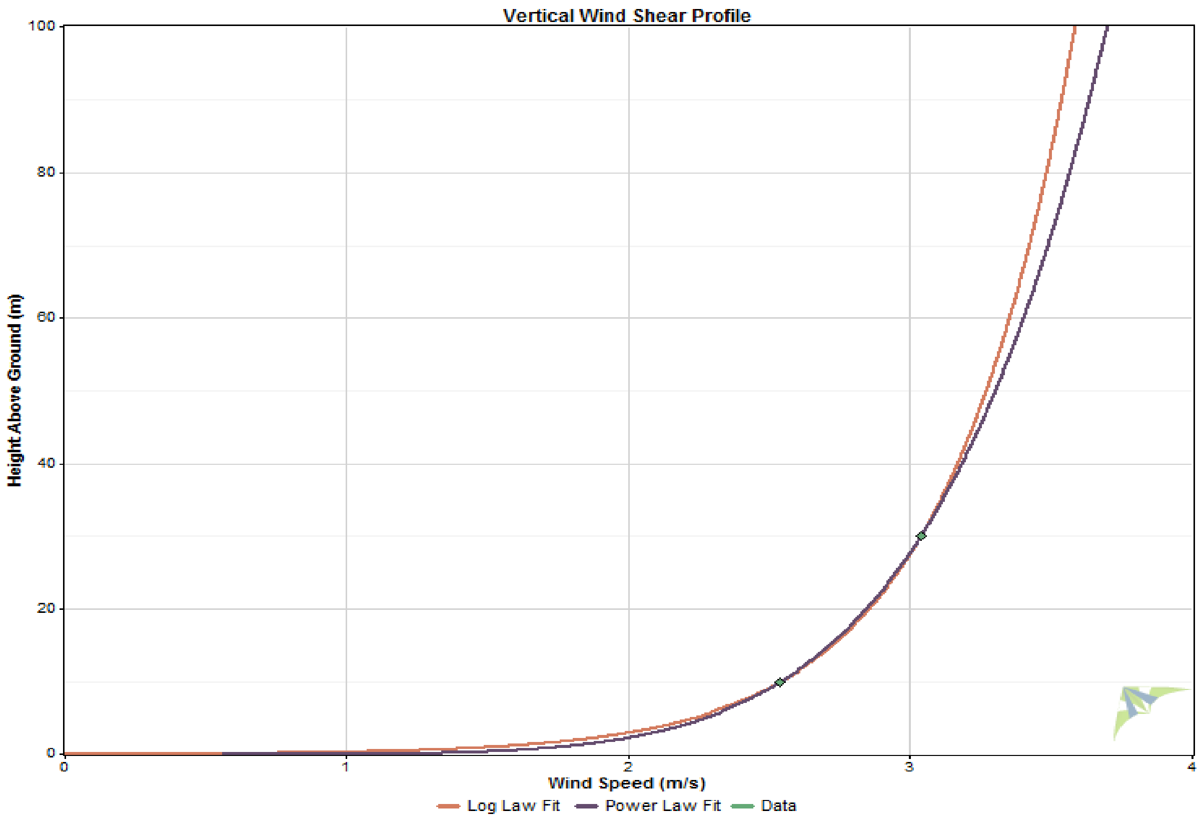

Figure 2; Wind shear profile

3.1 Diurnal Variation Of Average Wind Speeds

Diurnal variation of average wind speeds was obtained by use of MS Excel and was as per figure 3.

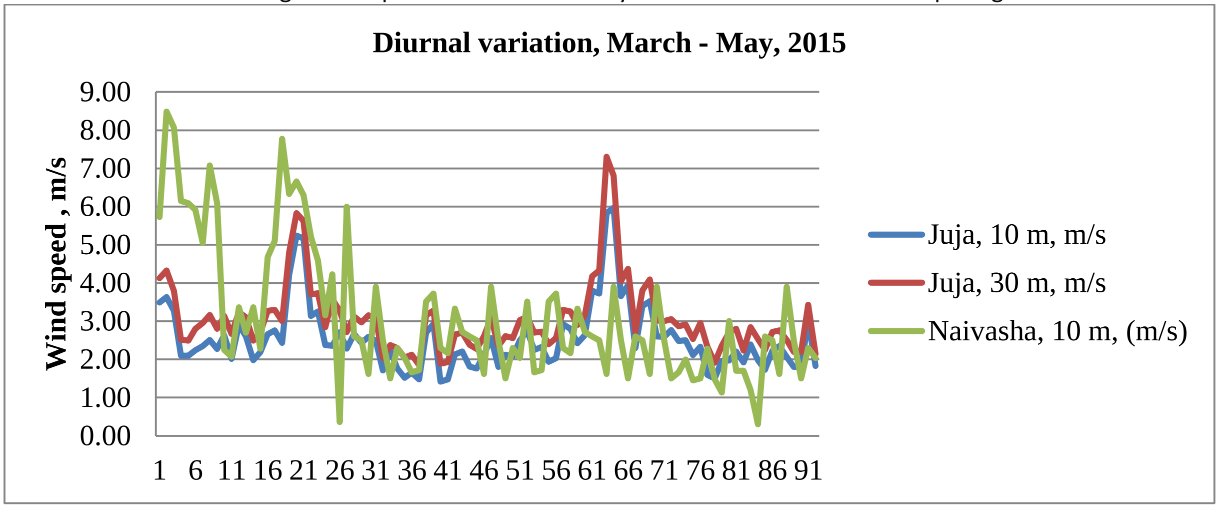

Figure 3: Diurnal variation, March - May, 2015

The two sites had low wind speeds of wind class 1, though for March and beginning of May was slightly higher. This can be attributed to higher temperatures which led to increase in pressure gradient. The diurnal profile portrays higher wind speeds in Naivasha than Juja site. Since the wind flow patterns are modified by the earth's terrain, bodies of water and vegetative cover because they determine the degree of roughness, Kargieve *et al*. (2001) and Saoke (2011). The slightly higher Naivasha winds can be attributed to the effect of the lake Naivasha and channeling effect of the Rift valley and the hills.

3.2 Diurnal Variation of Average Wind Directions

WindRose diagrams were used to analyze wind direction the overall one is shown by figure 4. It was generated using Windographer. Most Juja wind were between east north east (ENE) and east south east (ESE) while that of Naivasha was between south east (SE) and west (W).

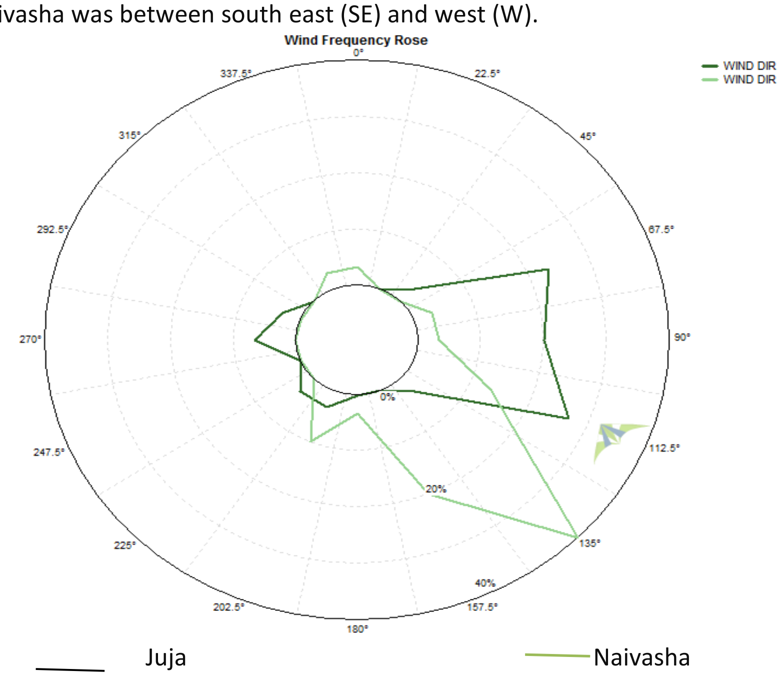

Figure 4: Overall WindRose, March- May

3.3 Frequency Distribution Functions per Power Density (FDFs)

Frequency Distribution Functions obtained by Windographer were as per figures 5, 6 and 7.

The FDFs show most average power densities were quite low. That was as a result of low wind speeds which were mainly between 2 m/s to 3 m/s.

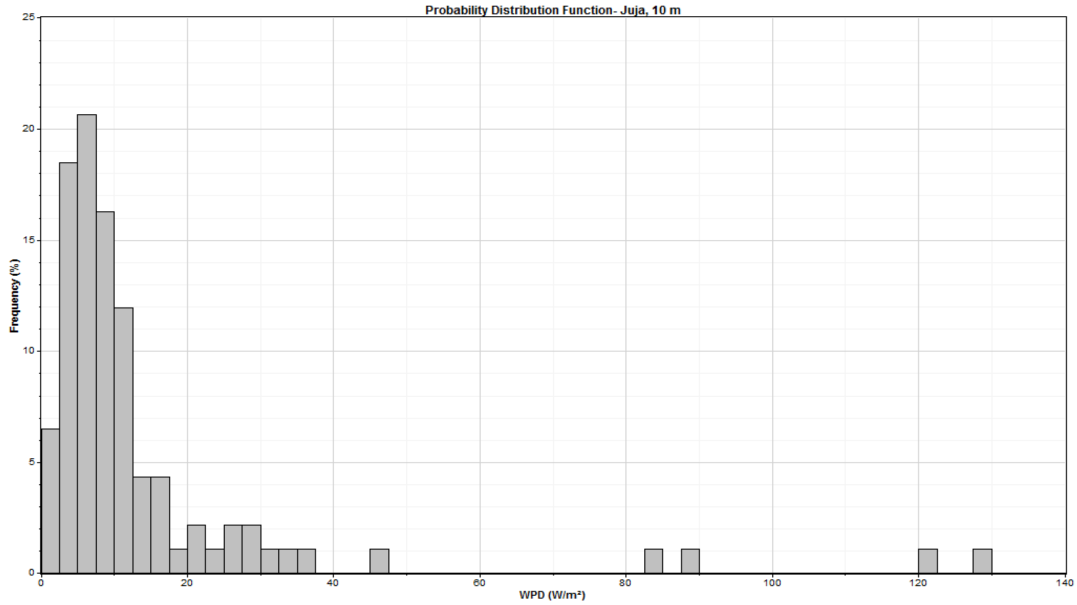

Figure 5: Frequency Distribution Functions (FDF) s, 10 m, Juja

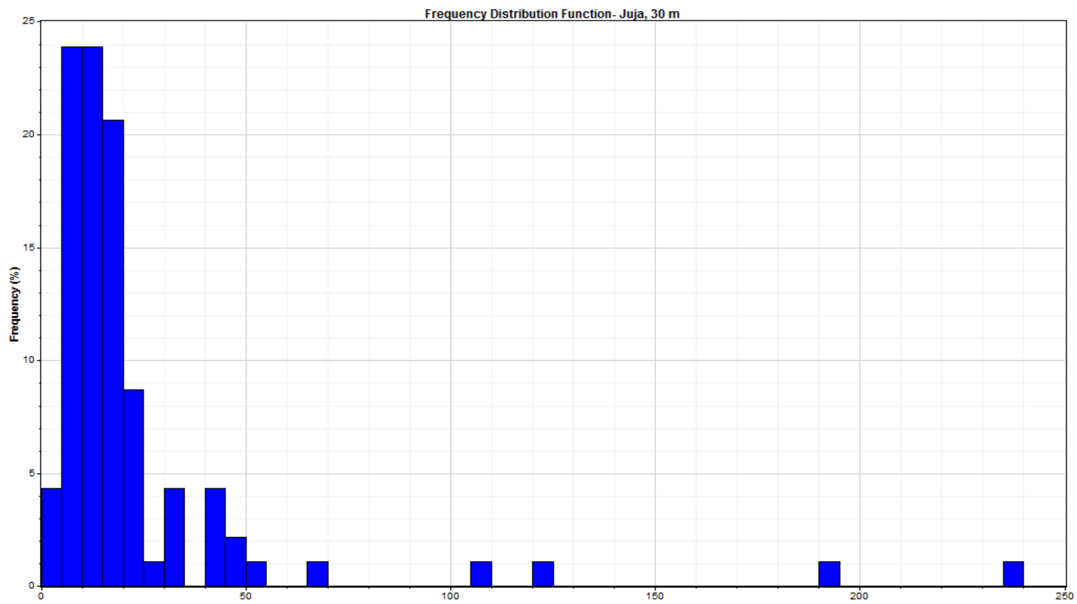

Figure 6: Frequency Distribution Functions (FDF), 30 m, Juja

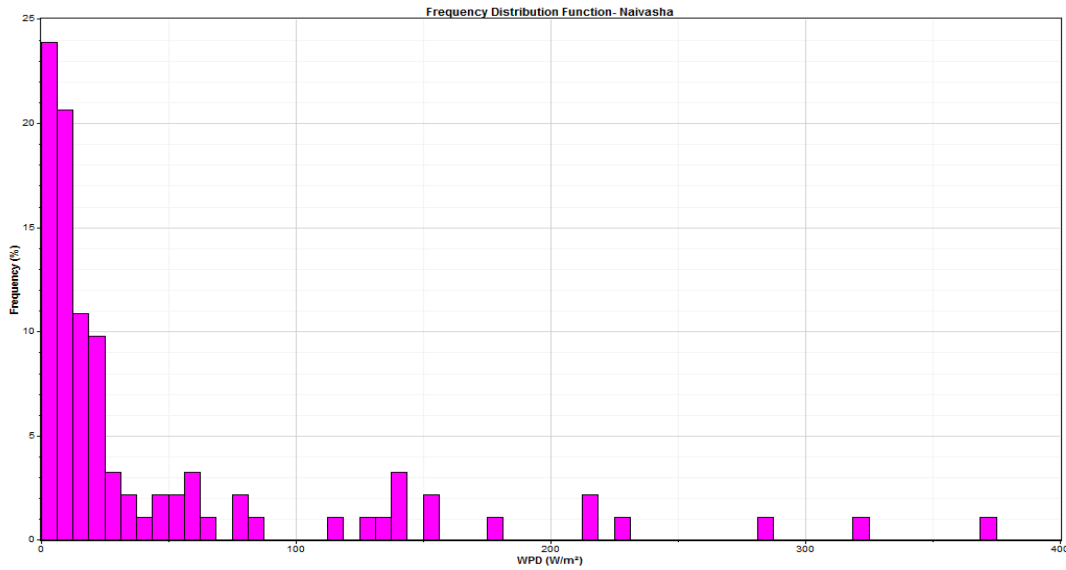

Figure 7: Frequency Distribution Functions (FDF) s –Naivasha

3.4 Weibull Parameters and Power Densities

The wind speeds were used to determine Weibull shape parameters (k), scale parameters (c), (table 1), and wind power densities, (table 2) for different methods. Windographer and Microsoft Excel were used to determine and generate the parameters and Probability Distribution Functions (figures 8, 9, and 10). .Actual power densities for 10 m, Juja, 30 m, Juja and 10 m, Naivasha were 14.5 W/m², 30.9 W/m² and 58.5 W/m² by Weibull model and 2.23 W/m², 2.73 W/m² and 2.83 W/m² by Rayleigh model respectively. The mean wind power densities for Juja and Naivasha for 10 m were 12.96 W/m² and 36.52 W/m² by Weibull and 2.82 W/m² and 3.31 W/m² for Rayleigh distribution function. Error analysis was done using equation 18 below.

$$Error(\%) = \frac{P_{(W,R)} - P_{(M,R)}}{P_{(M,R)}} \quad \text{..........}18$$

Where $P_{(W,R)}$ in (W/m^2) is the mean power density calculated from either the Weibull or Rayleigh function used in the calculation of the error and $P_{(M,R)}$ is the wind power density for probability density distribution derived from field data values which serve as the reference mean power density. The results on power density shows Weibull-fit is the method of best fit for Juja and MLH for Naivasha at 10 m by Weibull distribution model while Regression fits Juja best and Weibull-fit Naivasha by Rayleigh model. This implies that power density for any selected site should be determined using different methods before assigning it a class.

3.5 Probability Distribution Functions (PDFs)

The Probability Distribution Functions generated from Weibull parameters obtained from different methods are as per figures 8, 9 and 10. The area under each curve represents power density.

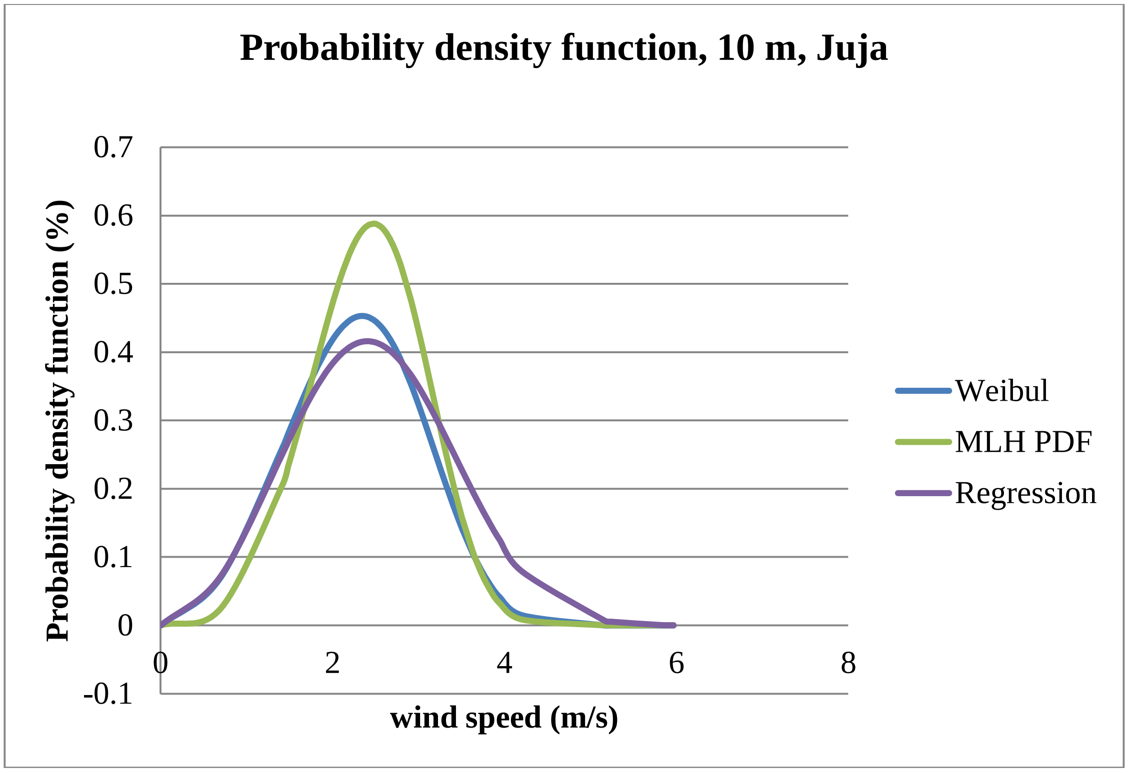

Figure 8: Probability Distribution Functions (PDF) s, 10 m, Juja

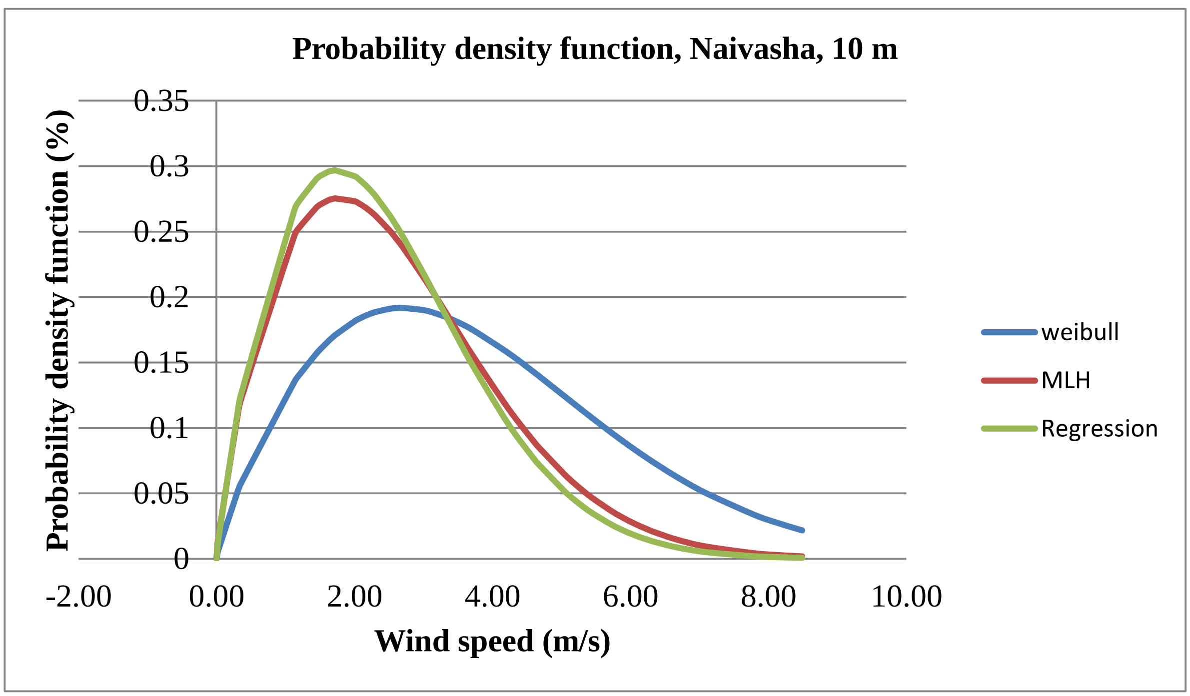

Figure 9: Probability Distribution Functions (PDF) s, 10 m, Naivasha

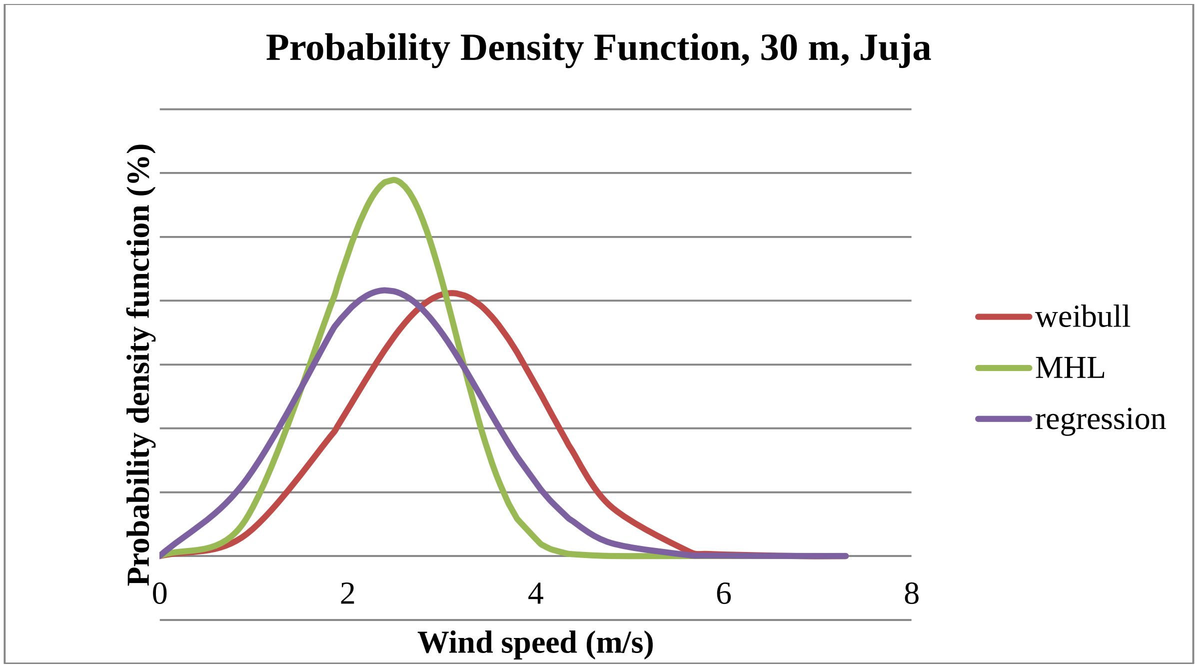

Figure 10: Probability Distribution Functions (PDF) s, 30 m, Juja

4.0 Conclusions

The following conclusions were made from wind data obtained from the two sites;

- The sites have low wind speed averages raging from 0.3 m/s to 3.54 m/s which resulted to low power density of wind class 1.
- The Weibull scale parameters c and shape parameters k ranged from 2.394 m/s to 4.209 m/s and 0.943 to 2.937 respectively.
- The mean wind power densities for Juja and Naivasha for 10 m were 12.96 W/m² and 36.52 W/m² by Weibull model and 2.82 W/m² and 3.31 W/m² by Rayleigh model.
- Actual power densities for 10 m, Juja, 30 m, Juja and 10 m, Naivasha were 14.5 W/m², 30.9 W/m² and 58.5 W/m² by Weibull model and 2.23 W/m², 2.73 W/m² and 2.83 W/m² by Rayleigh model respectively.
- Power density depends on method of analysis.
- Method of best fit is dependent on locality/ altitude as per Paitoon, 2010s' findings.

Acknowledgements

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References

Ajayi O. O. Fabenle R.O. and Katende J. (2011).Wind profile **c**haracteristics and econometrics analysis of wind power generation of a site in Sokoto state, Nigeria. *Energy Science and Technology*, **1:** pp54-66

Akpinar E. K. and Akpinar S. (2004). Statistical analysis of wind energy potential on the basis of Weibull and Rayleigh distribution for Agin-Elazig, Turkey. Proceedings of the Institution of Mechanical Engineer's. *Journal on Power and Energy*. **213**: pp557-563.

Justus C. G. Hargraves W.R. Mikhail A. and Graber D. (1978). Methods of estimating wind speed frequency distributions. *Journal of Applied Meteorology* **17**: pp17-18

Kamau J. N. Kinyua R. and Gathua J. K. (2009). Determination of the wind power parameters by statistical analysis, the case of Garissa; the Second scientific conference proceedings **2**(1): pp44-55

Kargieve V. M. Martirosov S.N. Murugov V. P. Pinov A.B., Sokolsky A.K., and Haritonov V.P., (2001*). Wind power engineering, an application guide for wind mills of small and average power*. Intresolar Centre, Moscow.

Kollu R. Rao S. Rayapudi M. Narasimham S. and Pakkurthi K.M. (2012). Mixture probability distribution functions to model wind speed distributions. *International Journal of Energy and Environmental Engineering* **27**: pp2251-6832

Linacre E. and Geerts B. (1999). *Roughness length*. Available at:
http//www-das.uwyo.edu/~geerts/cwx/notes/chap14/roughness.

Manwell J. F. McGowan J.G., and Rogers A.L. (2009). *Wind energy explained Theory, design and application*, John Wiley and sons Ltd. West Sussex, United Kingdom.

Paitoon S. (2010). *Demonstrating Measure-Correlate-predict algorithms for Estimation of Wind Resources in Central Finland;* MSc. thesis, University of Jyvaskyla, Finland

Piefrancesco P. (2011). Energy and Environment. *Internal Journal of energy and environment Engineering* **4**(40095): pp2008-9163

Saoke C. O. (2011). *Analysis of wind speeds on Weibull and data correlation for wind pattern description for a selected site in Juja*; M.Sc. thesis, Jomo Kenyatta University of Agriculture and Technology, Juja, Kenya

Kantar Y. N. and Senoglu B. (2008). A comparative study for the location and scale parameters of the Weibull distribution with given shape parameter. *Comput Geoscience* **34:** pp1900-9

Kantar Y. M and Usta I. (2008). Analysis of wind speed distributions function derived from minimum cross entropy principles as better alternative to Weibull function. *Energy conversion management* **49**: pp962-73

Zhang L. F. et al. (2006). Robust regression using probability plots for estimating the Weibull shape parameter. *Quality reliable Engineering International* **22**: pp905-917.

Zhou J. Erdem E. Li G. and Shi J. (2009).Comprehensive evaluation of wind speed distribution models: A case study for North Dakota sites. *Journal of Applied Meteorology* **51**(7): pp1449-1458

Appendices

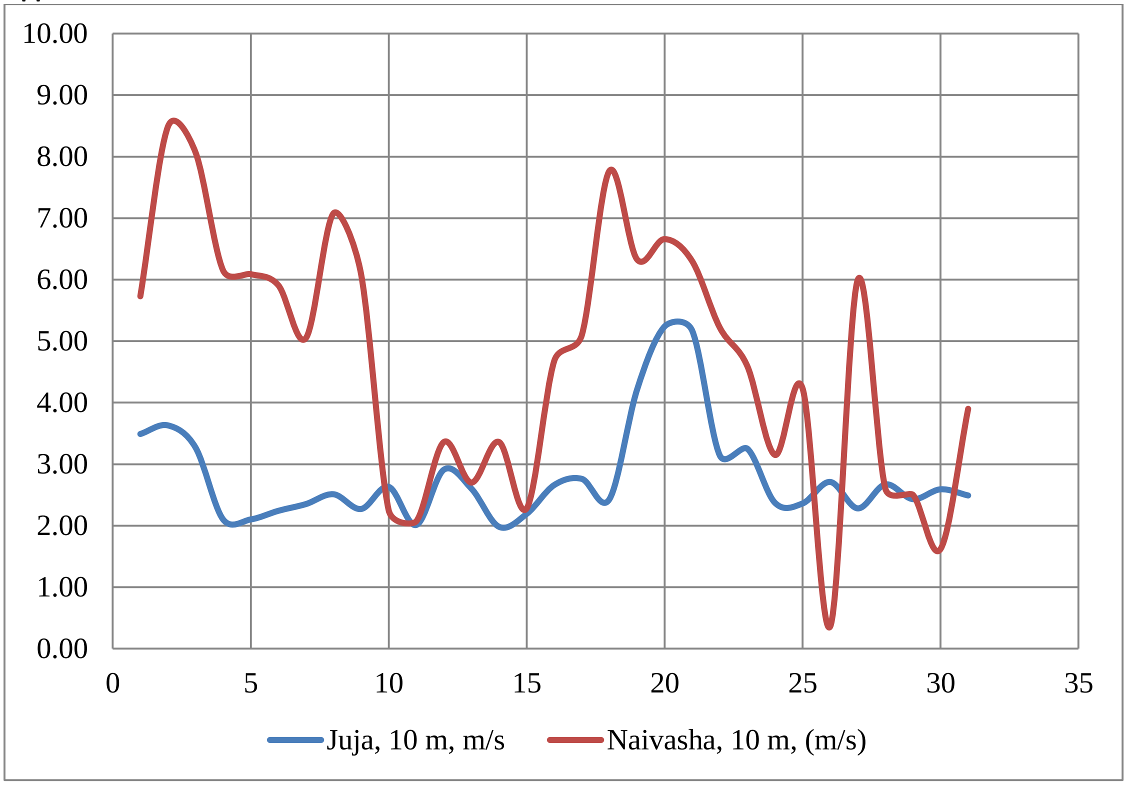

Figure 11: Diurnal variations, March, 2015

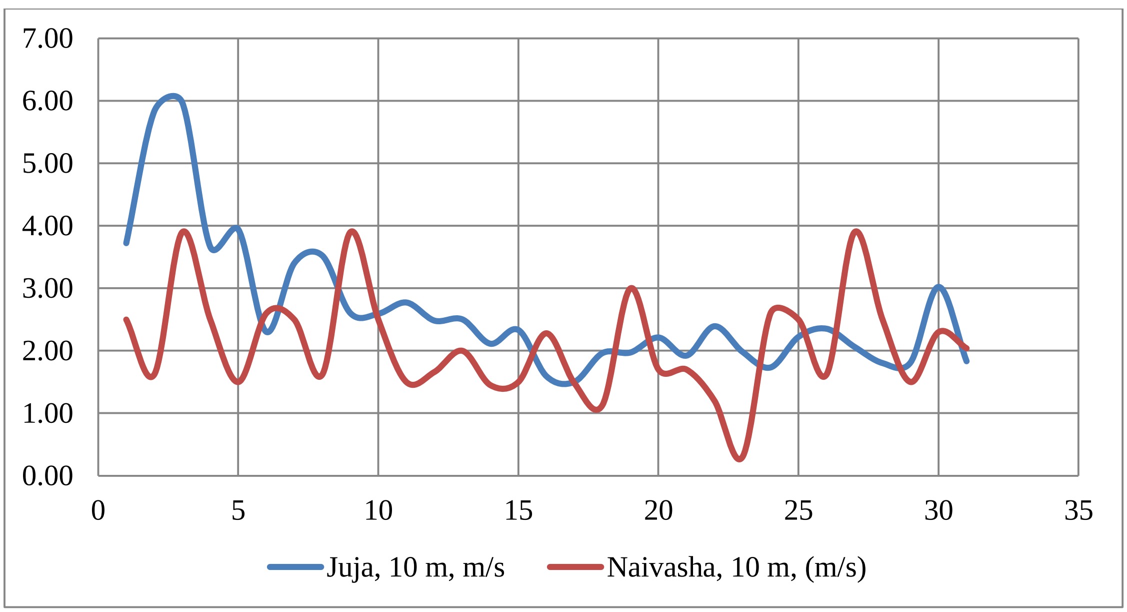

Figure 12: Diurnal variations, April, 2015

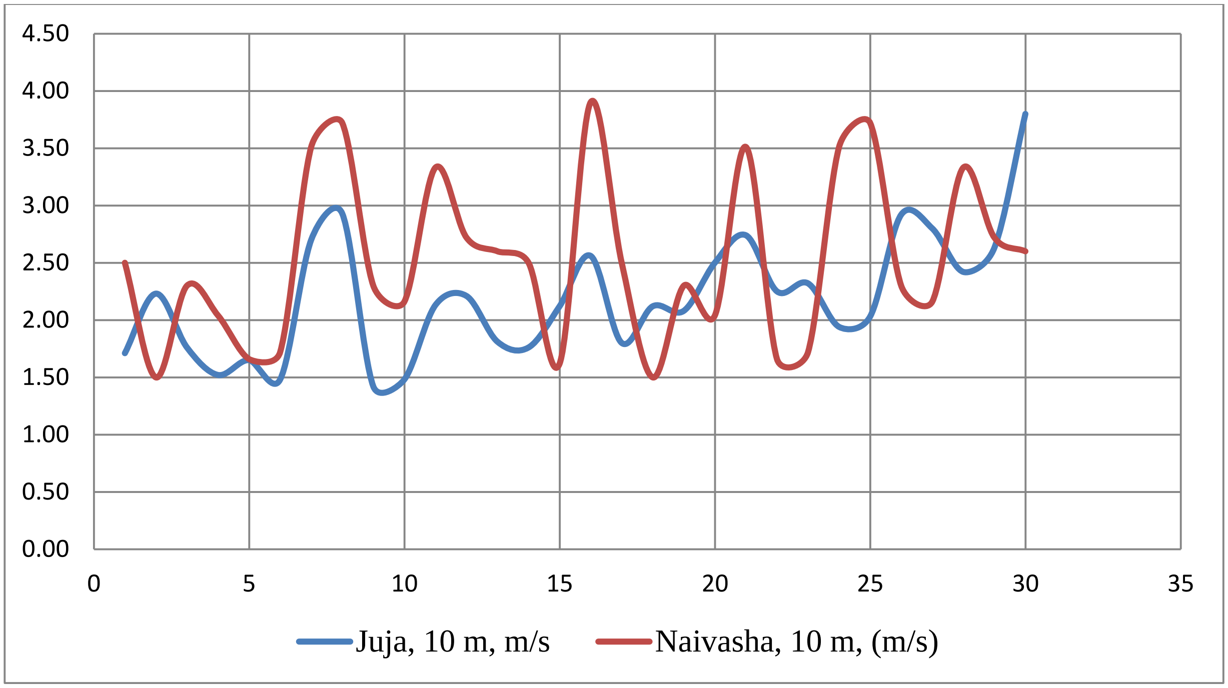

Figure 13: Diurnal variations, May, 2015

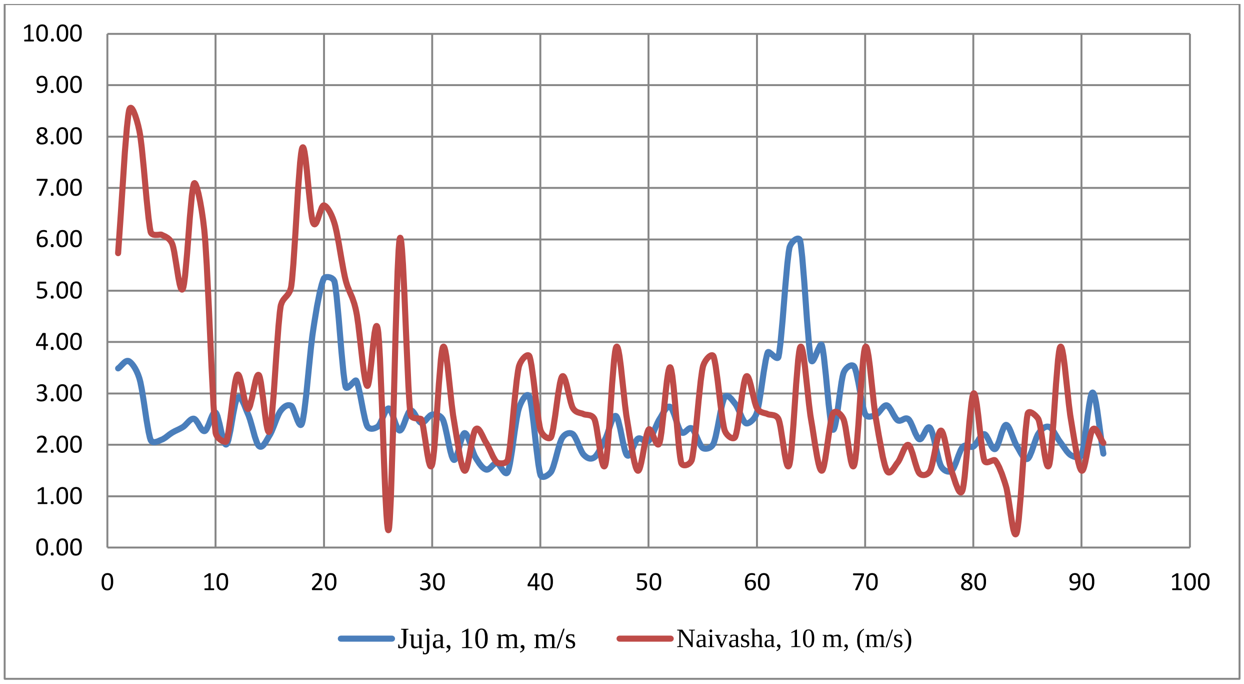

Figure 14: Diurnal variation of wind speeds, March-May, 2015

Table 1: Weibull parameters for different methods from Juja site and Naivasha

Method	At a height of 30 m (Juja site)		At a height of 30 m (Juja site)		Naivasha (St. Xavier)	
	c (m/s)	k	c (m/s)	k	c (m/s)	k
Weibull-fit	2.811	2.937	3.646	3.394	4.209	2.390
Regression	2.937	2.773	2.937	2.773	2.718	1.810
MLH	2.394	1.261	2.652	1.261	2.635	0.943

Table 2: Wind power densities

Month	Wind Power Density (W/m²)											
	At 10 m (Juja site)				At 30 m (Juja Site)				At 10 m (Naivasha site)			
	W	E_W	R	E_R	W	E_W	R	E_R	W	E_W	R	E_R
Weibull	13.81	0.05	2.92	-0.24	22.44	0.38	3.78	-0.28	68.34	-0.14	3.56	-0.29
Regression	13.73	0.06	3.05	-0.25	18.81	0.64	3.05	-0.11	18.42	0.68	2.30	0.23
MLH	10.5	0.38	2.91	-0.23	20.70	0.49	2.91	-0.06	84.90	-0.31	2.44	0.16

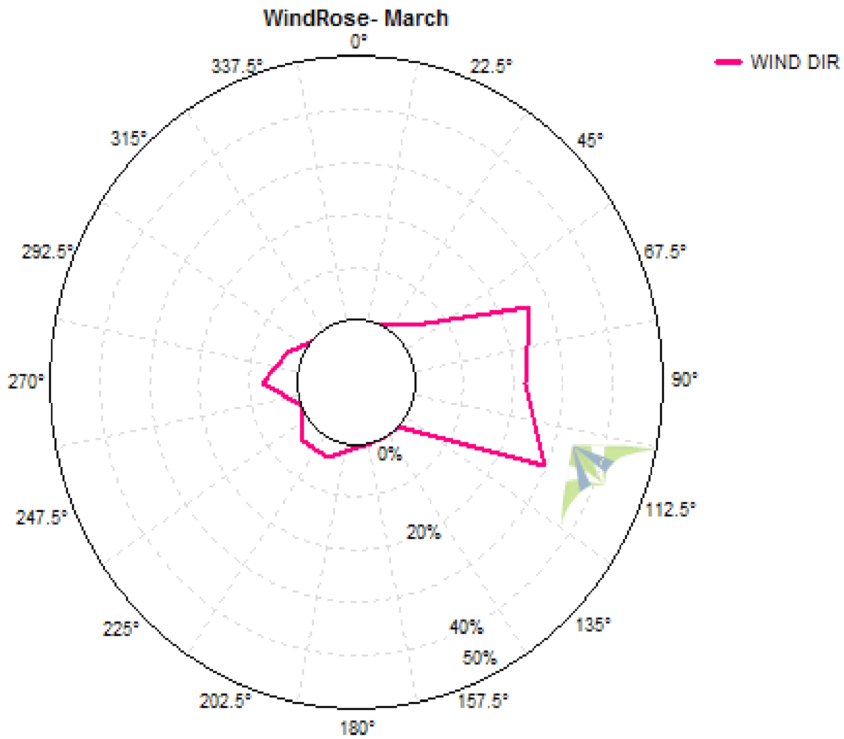

Figure 15: WindRose for Juja, March, 2015

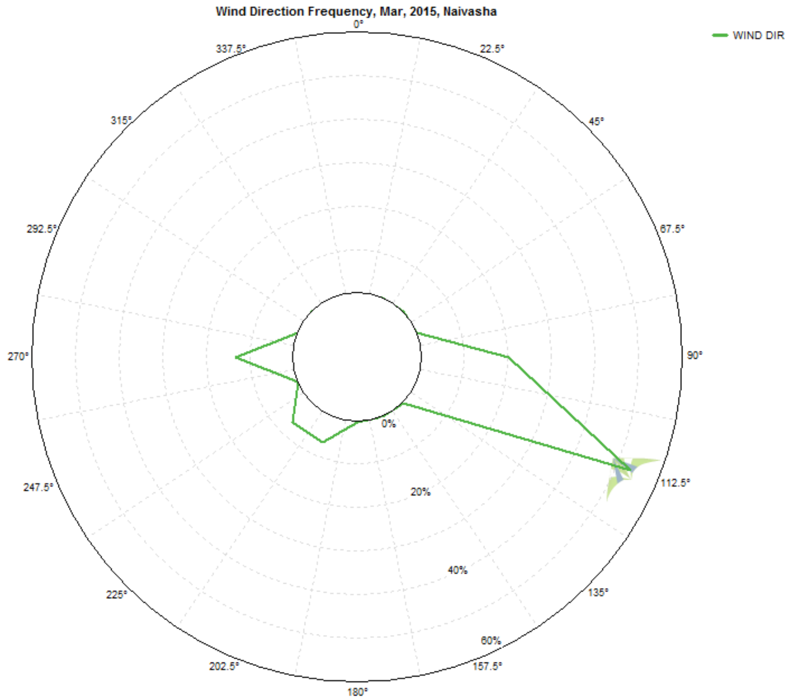

Figure 16: WindRose, Naivasha site, March, 2015

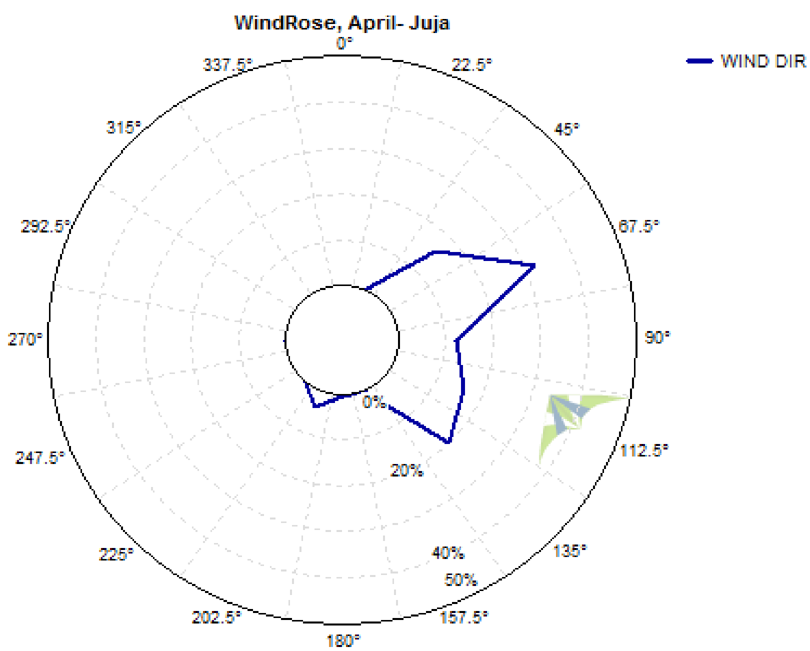

Figure 17: WindRose, Juja site, April, 2015

Figure 18: WindRose, Naivasha site, April, 2015

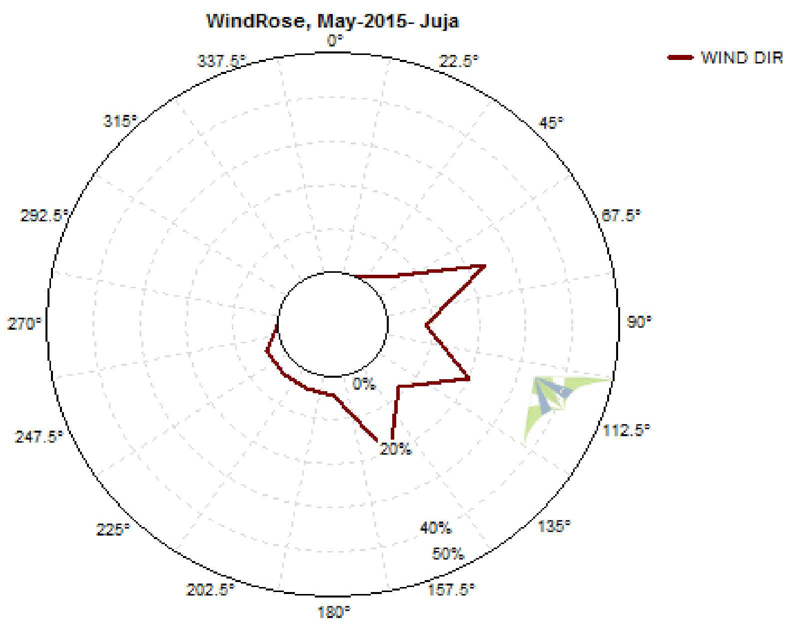

Figure 19: WindRose for Juja, May, 2015

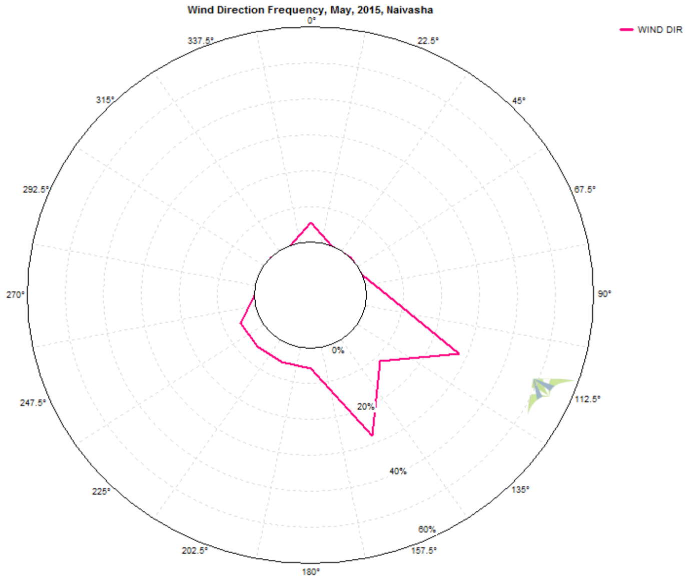

Figure 20: WindRose for Naivasha, May, 2015